Chapter 8 - Brief Notes

Sections 8.1 and 8.2 combine the concepts in Chapters 4 and 5.

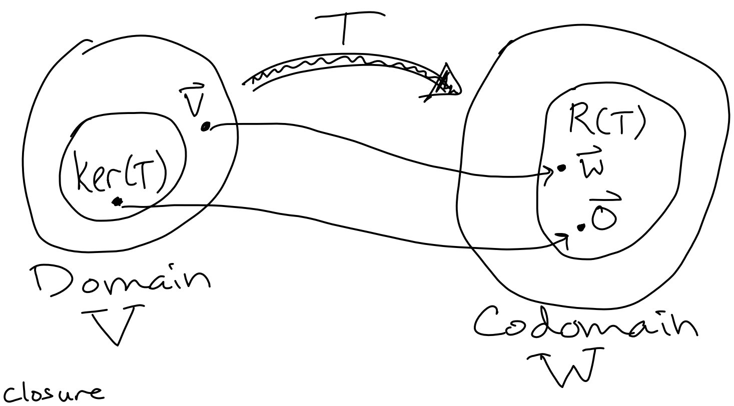
$T: V \to W$ is a linear transformation if $\vec{u}, \vec{v} \in V$

ⓐ $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ and ⓑ $T(c\vec{u}) = cT(\vec{u})$

V and W are no longer limited to $\mathbb{R}^n$.

Kernel of $T = \ker(T)$ is the set of all vectors in V that map to $\vec{0}$.

Range of $T = R(T)$ is the set of all vectors in W that get mapped to. (all of the images in W)



Example: Consider $T: \mathbb{R}^3 \to \mathbb{R}^3$ where T is orth. proj. onto the xy-plane.
What is $\ker(T)$? $R(T)$?

prove w/ closure and ⓐ + ⓑ above.

$\ker(T)$ is a subspace of V.

Dimension of kernel = nullity of T

If $\vec{v} \in \ker(T)$, then $T(\vec{v}) = \vec{0}$

like $A\vec{x} = \vec{0}$ when $T(\vec{x}) = A\vec{x}$

prove w/ closure and ⓐ + ⓑ above.

$R(T)$ is a subspace of W.

Dimension of range = rank of T

If $\vec{v} \in V$ and $T(\vec{v}) = \vec{w}$,
then $\vec{w} \in R(T)$ ← set of all $\vec{w}$'s

like $A\vec{x} = \vec{b}$ when $T(\vec{x}) = A\vec{x}$

Kernel ~ Nullspace

set of all $\vec{v}$ such that $T(\vec{v}) = \vec{0}$

set of all $\vec{x}$ such that $A\vec{x} = \vec{0}$

Range ~ Column Space

Set of all $\vec{w}$ such that there is a $\vec{v}$ where $T(\vec{v}) = \vec{w}$

Set of all $\vec{b}$ such that there is a $\vec{x}$ where $A\vec{x} = \vec{b}$

$$\underset{A}{\begin{bmatrix} \vec{c}_1 & \vec{c}_2 & \vec{c}_3 \end{bmatrix}} \underset{\vec{x}}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{\vec{b}}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}$$

$x_1\vec{c}_1 + x_2\vec{c}_2 + x_3\vec{c}_3 = \vec{b}$

span of col. vectors = col. space

The set of all $\vec{b}$'s is the column space

rank + nullity = dimension of domain

Ex: Orth. proj. onto xy-plane. What is the rank? nullity?

Ch. 4: # of columns of A

Ex: $T: \mathbb{R}^5 \to \mathbb{R}^3$

$$A = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

need $\vec{x}$ to be 5×1

$\vec{x} \in \mathbb{R}^5$

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