Quiz 15.1 - 15.4 ______ Name: Key
4 pts. per problem #1 - 4. 18 pts. Per.: ______
No calculators.

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1) Evaluate the integral $\int_0^{\sqrt{\pi}} \int_{\pi/6}^{y^2} 2y\cos x \, dxdy$.

$$\int_0^{\sqrt{\pi}} 2y\sin x \Big|_{\pi/6}^{y^2} dy$$

$$\int_0^{\sqrt{\pi}} 2y\sin y^2 - 2\frac{y}{2}\, dy$$

$u = y^2 \quad du = 2y$

$$-\cos u \Big|_0^{\pi} - \frac{y^2}{2}\Big|_0^{\sqrt{\pi}}$$

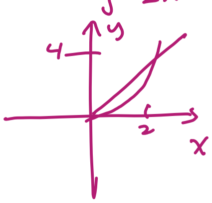
$$-(-1) - (-1) - \left(\frac{\pi}{2}\right)$$

$$\boxed{2 - \frac{\pi}{2}}$$

2) Evaluate $\int_0^4 \int_{y/2}^{\sqrt{y}} (x^2 + 4y)\, dxdy$ after switching the order of integration.

$x = \sqrt{y}$

$y = 2x$



$$\int_0^2 \int_x^{2x} (x^2 + 4y)\, dydx$$

$$\int_0^2 x^2y + 2y^2 \Big|_{x^2}^{2x} dx$$

$$\int_0^2 2x^3 + 8x^2 - x^4 - 2x^4 \, dx$$

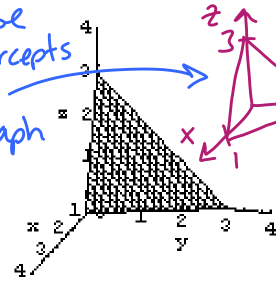
$$\frac{1}{2}x^4 + \frac{8}{3}x^3 - \frac{3}{5}x^5 \Big|_0^2$$

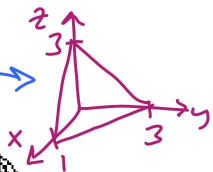
$$8 + \frac{64}{3} - \frac{96}{5} - 0$$

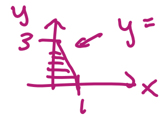
$$\boxed{\frac{152}{15}}$$

3) Use a *double integral* to find the volume of the solid bounded by the plane $3x + y + z = 3$ in the first octant.

Use intercepts to graph







$y = -3x + 3$

$z = 3 - 3x - y$

$$\int_0^1 \int_0^{-3x+3} (3 - 3x - y)\, dydx$$

$$\int_0^1 3y - 3xy - \frac{1}{2}y^2 \Big|_0^{-3x+3} dx$$

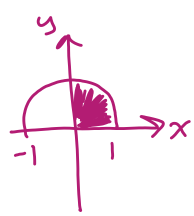
$$\int_0^1 -9x + 9 + 9x^2 - 9x - \frac{9x^2}{2} + \frac{18x}{2} - \frac{9}{2}\, dx$$

$$\int_0^1 \frac{9x^2}{2} - \frac{18x}{2} + \frac{9}{2}\, dx$$

$$\frac{3}{2}x^3 - \frac{9}{2}x^2 + \frac{9}{2}x \Big|_0^1$$

$$\frac{3}{2} - \frac{9}{2} + \frac{9}{2} = \boxed{\frac{3}{2}}$$

4) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}\, dydx$ using polar coordinates. Sketch R.



$$\int_0^{\pi/2} \int_0^1 e^{-r^2}\, r\,dr\,d\theta$$

$u = -r^2$
$du = -2r\,dr$

$$-\frac{1}{2}\int_0^{\pi/2} \int_0^{-1} e^u\, du\, d\theta$$

$$-\frac{1}{2}\int_0^{\pi/2} e^u \Big|_0^{-1} d\theta$$

$$-\frac{1}{2}\int_0^{\pi/2} e^{-1} - 1\, d\theta$$

$$\boxed{-\frac{\pi}{4}\left(\frac{1}{e} - 1\right)}$$

From 15.5 reading. Not tested.

Flat surface that has mass.

5) True or False? If ρ is a continuous density function on the lamina corresponding to a plane region R, then the mass m of the lamina is given by $m = \iint_R \rho(x, y)\, dA$. True (2 pts.)