Students are expected to complete homework assignments on their own before referring to the following pages. The answers and hints are designed to check work and clarify problems. The original intent of the layout was for display in class after assignments had been completed. Students should use the following information as help to understand the exercises and master the concepts.

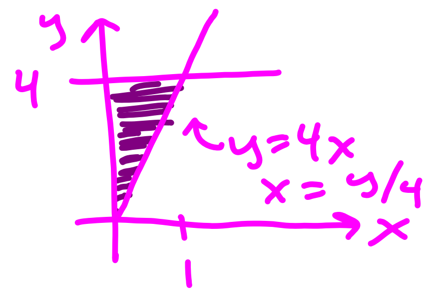
Calculus D

Chapter 15

Even Answers & Hints for Homework

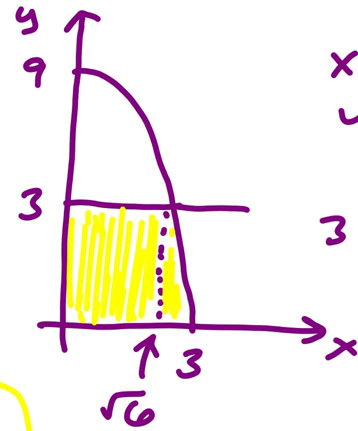
15.3 Even Answers

40 $\int_0^1 \int_{4x}^4 f(x,y)\,dy\,dx$



$$= \boxed{\int_0^4 \int_0^{y/4} f(x,y)\,dx\,dy}$$

42 $\int_0^3 \int_0^{\sqrt{9-y}} f(x,y)\,dx\,dy$



$x=\sqrt{9-y}$

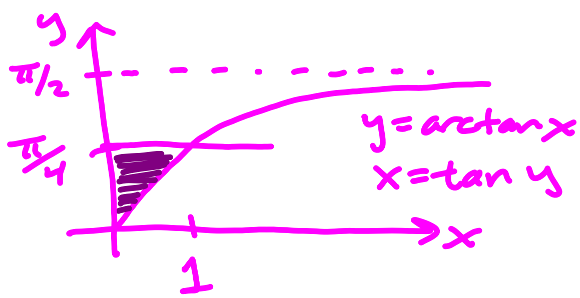
$y=9-x^2$

$3=9-x^2$

$x=\pm\sqrt{6}$

$$= \boxed{\int_0^{\sqrt{6}} \int_0^3 f(x,y)\,dy\,dx + \int_{\sqrt{6}}^3 \int_0^{9-x^2} f(x,y)\,dy\,dx}$$

44 $\int_0^1 \int_{\arctan x}^{\pi/4} f(x,y)\,dy\,dx$



$$= \boxed{\int_0^{\pi/4} \int_0^{\tan y} f(x,y)\,dx\,dy}$$

46 $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\,dx\,dy = \int_0^{\sqrt{\pi}} \int_0^x \cos(x^2)\,dy\,dx = \boxed{0}$

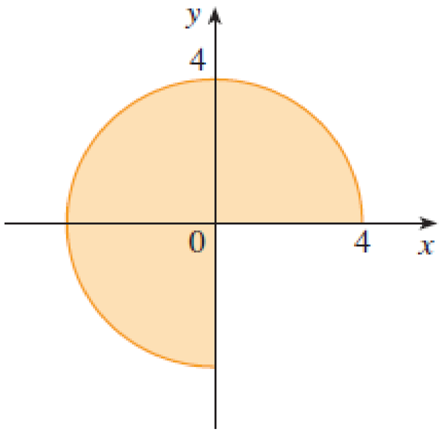
48 $\int_0^1 \int_x^1 e^{x/y}\,dy\,dx = \int_0^1 \int_0^y e^{x/y}\,dx\,dy = \boxed{\frac{1}{2}(e-1)}$

50 $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\,dx\,dy = \int_0^2 \int_0^{x^3} e^{x^4}\,dy\,dx = \boxed{\frac{1}{4}(e^{16}-1)}$

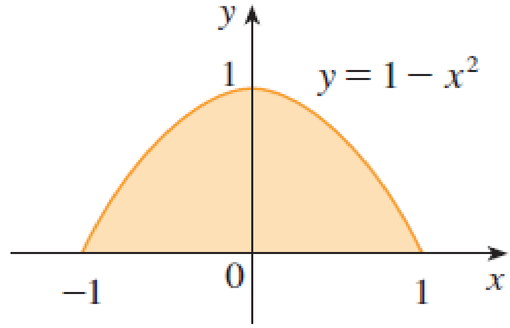
15.4 Even Answers

1–4 A region R is shown. Decide whether to use polar coordinates or rectangular coordinates and write $\iint_R f(x, y)\, dA$ as an iterated integral, where f is an arbitrary continuous function on R.

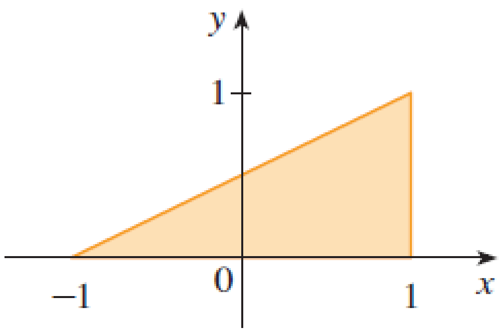
1.



2.



3.



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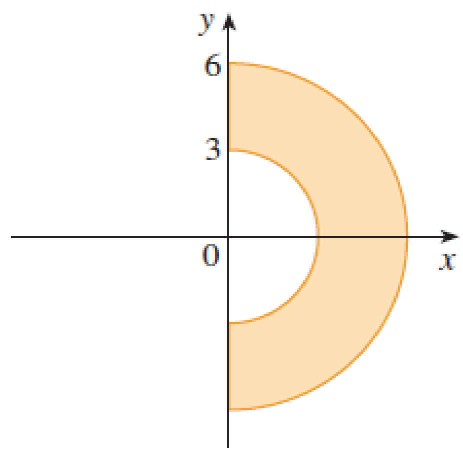
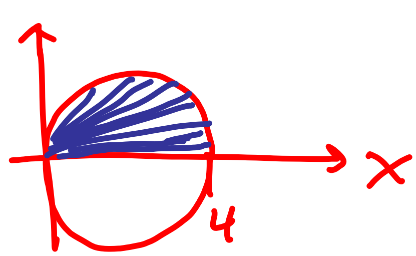


(2) Rectangular: $\int_{-1}^{1} \int_{0}^{1-x^2} f(x,y)\, dy\, dx$

(4) Polar: $\int_{-\frac{\pi}{2}}^{\pi/2} \int_{3}^{6} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$

(6) $\int_{0}^{\pi/2} \int_{0}^{4\cos\theta} r\, dr\, d\theta$

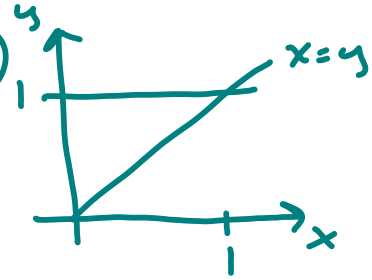
$= 2\pi$



15.6 (Stewart 4th ed.)

Some Surface Area Setups

(4)



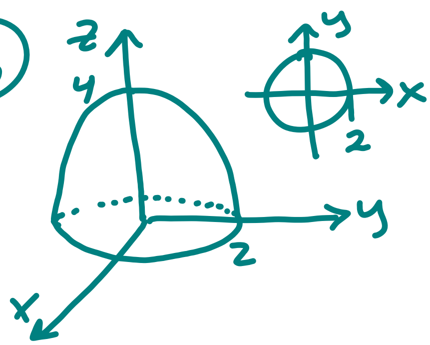
$$S = \int_0^1 \int_0^y \sqrt{2+4y^2}\, dx\, dy$$

$$= \frac{3}{\sqrt{6}} - \frac{1}{3\sqrt{2}}$$

(5) $S = \int_0^4 \int_0^2 \frac{3}{\sqrt{9-y^2}}\, dy\, dx = 12 \arcsin \frac{2}{3}$

$\arcsin \frac{y}{3}$

(6)

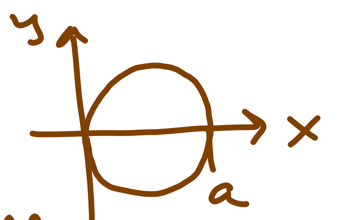


$$S = \iint_R \sqrt{1+(-2x)^2+(-2y)^2}\, dA$$

$$= \int_0^{2\pi} \int_0^2 \sqrt{1+4r^2}\; r\, dr\, d\theta$$

$$= \frac{\pi}{6}(17\sqrt{17}-1)$$

(11) $z = \sqrt{a^2-x^2-y^2}$



$$S = \iint_R \sqrt{\frac{x^2+y^2}{a^2-x^2-y^2}+1}\, dA$$

$$= \int_{-\pi/2}^{\pi/2} \int_0^{a\cos\theta} \sqrt{\frac{r^2}{a^2-r^2}+1}\; r\, dr\, d\theta$$

(15) $S = \int_0^1 \int_0^1 \sqrt{4x^2+5}\, dy\, dx$

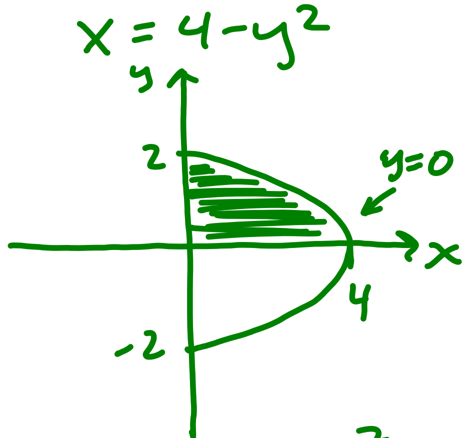
(17) $S = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{4x^2y^4+4x^4y^2+1}\, dy\, dx$

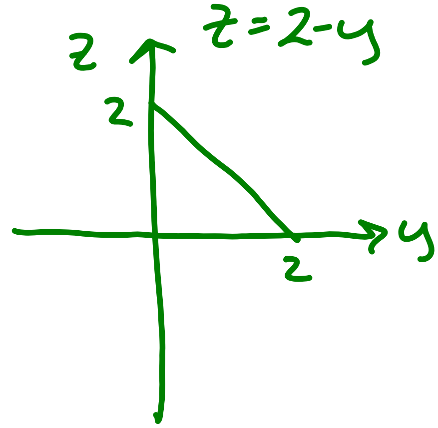
(21) Do "polar" in xz-plane.

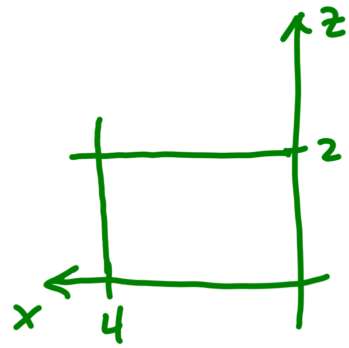
15.6 Even Answers

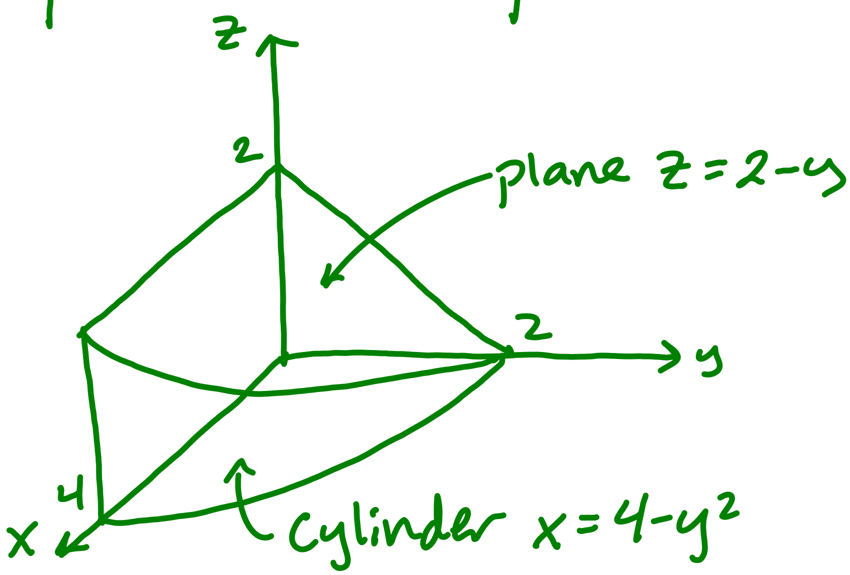
(12) $4/3$ (22) 128π

(28) $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\,dz\,dy$







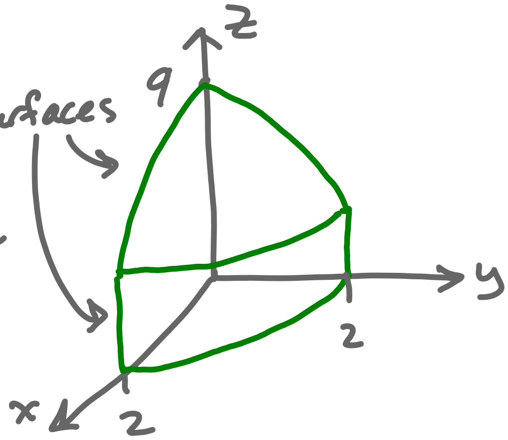


15.7 + 15.8 Even Answers

15.7

(16) $z = 9 - r^2$
$r = 2$ ← 2 surfaces

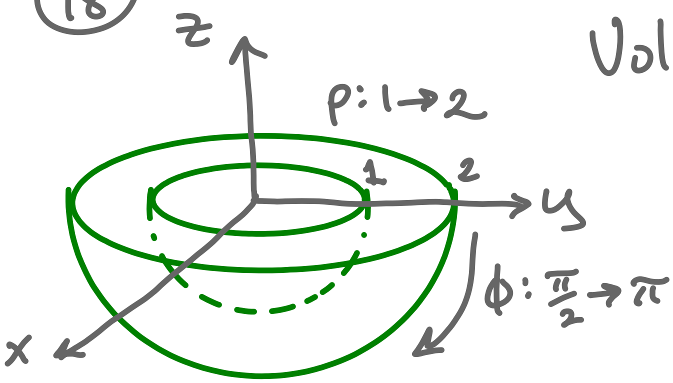
Volume $= 7\pi$



(28) $\frac{162}{5}\pi$

15.8

(18)



Volume $= \frac{14\pi}{3}$

(20) $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{2\pi} \int_1^2 f \, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$f(x,y,z)$ becomes

$f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$

(40) $\int_0^{\pi} \int_0^{2\pi} \int_0^{a} (\rho^3 \cos\phi) \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi = 0$