Quiz 10.1 - 10.3

2 points each. No calculator

Show work on separate paper.

12 pts.

Name: Key

Per: ____

SOLUTIONS ON NEXT PAGE

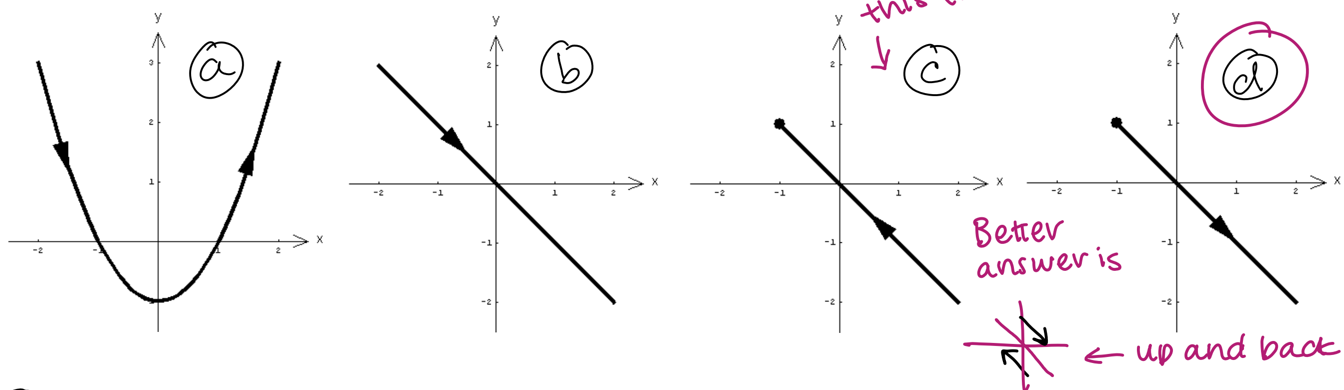
① Find the arc length of the curve: $x = t^2$, $y = 2t^2 + 1$, $1 \leq t \leq 3$.

ⓐ $16\sqrt{5}$ ⓑ 40 ⓒ 24 ⓓ $8\sqrt{5}$ ⓔ None of these

② Graph the curve given by the parametric equations $x = t^2 - 1$ and $y = 1 - t^2$.

If allow $t<0$, it does this first

ⓔ None of these



③ Find the corresponding rectangular equation by eliminating the parameter. $x = t^2 + 2$, $y = t^2 - 1$

ⓐ $x + y = 1$ ⓑ $y = x + 1$ ⓒ $x = y + 1$ ⓓ $y = x - 3$ with $x \geq 2$ ⓔ None of these

④ Find $\frac{d^2y}{dx^2}$ if $x = 2\cos\theta$, $y = \sin\theta$.

ⓐ $-\frac{1}{4}\csc^3\theta$ ⓑ $\frac{1}{2}\csc^2\theta$ ⓒ $-2\sec^2\theta$ ⓓ $\frac{1}{2}\cot\theta\csc\theta$ ⓔ None of these

⑤ Convert the rectangular equation $x^2 + y^2 - 2y = 0$ to polar form.

ⓐ $r = 2\cos\theta$ ⓑ $r = \frac{1}{2}\csc\theta$ ⓒ $r = 2\sin\theta$ ⓓ $r = -2\sin\theta$ ⓔ None of these

⑥ Find $\frac{dy}{dx}$ if $x = \sqrt{t}$ and $y = (t-1)^3$

ⓐ $3(t-1)^2$ ⓑ $\frac{1}{6\sqrt{t}(t-1)^2}$ ⓒ $\frac{6(t-1)^2}{\sqrt{t}}$ ⓓ $6\sqrt{t}(t-1)^2$ ⓔ None of these

Problms from Larson

Work Solutions

1. Arc length: $x=t^2,\ y=2t^2+1,\ 1\le t\le 3$

$$S=\int_a^b \sqrt{(x'(t))^2+(y'(t))^2}\,dt \qquad \begin{array}{l} x'(t)=2t \\ y'(t)=4t \end{array}$$

$$=\int^3 \sqrt{(2t)^2+(4t)^2}\,dt$$

$$=\int_1^3 \sqrt{4t^2+16t^2}\,dt$$

$$=\int_1^3 t\sqrt{20}\,dt$$

$$=\cancel{\tfrac{1}{2}}t^2\cdot\cancel{2}\sqrt{5}\,\Big|_1^3$$

$$=(9-1)\sqrt{5}$$

$$=\boxed{8\sqrt{5}}$$

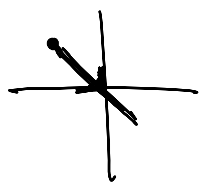
2. $x=t^2-1 \qquad -x=1-t^2$

$y=1-t^2 \rightarrow y=-x$

$y\le 1,\ x\ge -1$

$t=0 \Rightarrow (-1,1)$

$t=1 \Rightarrow (0,0)$



3. $x=t^2+2 \qquad y=t^2-1$

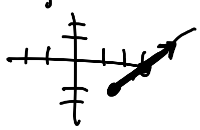
$t^2=x-2 \qquad \hookrightarrow y=(x-2)-1$

$y=x-3$

$t=0 \Rightarrow (2,-1)$

$t=1 \Rightarrow (3,0)$

$t=-1 \Rightarrow (3,0)$



$$\boxed{x\ge 2}$$

4. $x=2\cos\theta \qquad y=\sin\theta$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\cos\theta}{-2\sin\theta}=-\frac{1}{2}\cot\theta$$

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{d\theta}[dy/dx]}{dx/d\theta}=\frac{\frac{1}{2}\csc^2\theta}{-2\sin\theta}$$

$$\boxed{=-\frac{1}{4}\csc^3\theta}$$

5. $x^2+y^2-2y=0$

$r^2\cos^2\theta+r^2\sin^2\theta-2r\sin\theta=0$

$r^2-2r\sin\theta=0$

$r^2=2r\sin\theta$

$$\boxed{r=2\sin\theta}$$

6. $x=\sqrt{t}$

$y=(t-1)^3$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3(t-1)^2}{\frac{1}{2}t^{-1/2}}=\boxed{6\sqrt{t}(t-1)^2}$$