

Students are expected to complete homework assignments on their own before referring to the following pages. The answers and hints are designed to check work and clarify problems. The original intent of the layout was for display in class after assignments had been completed. Students should use the following information as help to understand the exercises and master the concepts.

Calculus C

Chapter 11

Even Answers & Hints
for Homework

11.1 Even Answers (Sequences)

② (a) Convergent Sequence \Rightarrow Limit Exists

Examples: _____, _____

(b) Divergent Sequence $\Rightarrow \lim_{n \rightarrow \infty} a_n$ does not exist

Examples: _____, _____

④ $a_n = \frac{n+1}{3n-1} \Rightarrow \left\{ \frac{2}{2}, \frac{3}{5}, \frac{4}{8}, \frac{5}{11}, \frac{6}{14}, \dots \right\}$

⑥ $a_n = 2 \cdot 4 \cdot 6 \cdots (2n) \Rightarrow \{2, 8, 48, 384, 3840, \dots\}$

⑧ $a_1 = 4$
 $a_{n+1} = \frac{a_n}{a_{n-1}} \Rightarrow \left\{ 4, \frac{4}{3}, 4, \frac{4}{3}, 4, \dots \right\}$

⑩ $\left\{ 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \dots \right\} \Rightarrow a_n = \frac{1}{3^{n-1}}$

⑫ $\left\{ -\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots \right\} \Rightarrow a_n = (-1)^n \frac{n}{(n+1)^2}$

⑭ $\{5, 1, 5, 1, 5, 1, \dots\} \Rightarrow a_n = 3 + 2(-1)^{n+1}$

11.2 Even Answers

$$(38) \sum_{n=1}^{\infty} \ln \frac{n}{n+1} \quad \text{note: } \ln \frac{n}{n+1} = \ln n - \ln(n+1)$$

$$S_n = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) \\ + \dots + (\ln n - \ln(n+1))$$

$$= \ln 1 - \ln(n+1) = -\ln(n+1)$$

$$\therefore \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} (-\ln(n+1)) = -\infty$$

\therefore diverges

11.3 Even Answers

- ④ Converges (Integral Test)
- ⑥ Diverges (Integral Test)
- ⑧ Diverges (Integral Test) Note: n^{th} term test works too
- ⑩ Converges (Two p-series)
- ⑫ Converges ($p = 3/2 > 1$)
- ⑭ Diverges (Integral Test $\sum \frac{1}{3n+2}$)
- ⑯ Diverges (Integral Test)
- ⑰ Diverges (Integral Test w/ Partial Fractions: $\frac{2}{x} + \frac{1}{x+1}$)
- ⑳ Converges (Integral Test)
- ㉔ Converges (Integral Test)

11.5 - HW Answers

- ③ $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{4}{n+6}$ Converges conditionally
- ⑤ Converges Conditionally (Limit Comp. to $\sum \frac{1}{n}$)
- ⑦ Diverges (n^{th} term test)
- ⑨ Converges Absolutely (Ratio Test)
- ⑪ Converges Conditionally (Limit Comp. to $\sum \frac{1}{n}$)
- ⑬ Diverges (n^{th} term test)
- ⑮ Converges Conditionally (p -series)
- ⑰ Converges Conditionally (Limit comp. to $\sum \frac{1}{n}$)
- ⑲ Diverges (n^{th} term test) $\frac{n^n}{n!} = \frac{n \cdot n \cdot n \cdots n}{1 \cdot 2 \cdot 3 \cdots n}$ ← bigger
← smaller

11.6 Even Answers

②0 Converges Absolutely (Root Test)

②2 Diverges (Root Test)

11.7 - Extra Practice - Even Answers (6th Ed.)

- ② Converges, Root Test
- ④ Converges, Alt. series, use $f'(x) < 0$, conditionally
- ⑥ Diverges, Limit Comparison or Integral Test
- ⑧ Diverges, Ratio Test
- ⑩ Converges, Integral Test $\frac{1}{3e}$
- ⑫ Diverges, n^{th} Term test
- ⑭ Converges absolutely, Direct Comparison $\sum |a_n|$ with $\sum (\frac{1}{2})^n$
- ⑯ Diverges, Limit Comparison
- ⑰ Converges, Alt. Series, Conditionally
- ⑲ Converges, Ratio Test
- ⑳ Converges, Direct Comparison ($\sum \frac{1}{n^2}$)
- ㉑ Diverges, n^{th} Term Test
- ㉓ Converges, Ratio Test
- ㉕ Converges, Direct Comparison to $\sum \frac{e}{n^2}$ or Integral Test
- ㉗ Converges, Alt. Series, Conditionally
- ㉙ Diverges, Root Test
- ㉛ Diverges, Direct Comparison with $\sum \frac{1}{2^n}$
- ㉝ Converges, Direct Comparison ($\sum \frac{1}{n^2}$)
- Note: $(\ln n)^{\ln n} = (e^{\ln \ln n})^{\ln n} = (e^{\ln n})^{\ln \ln n} = n^{\ln \ln n}$
- ㉟ Diverges, Limit Comparison with $\sum \frac{1}{n}$ + L'Hôpital's Rule

Note: Other tests may apply for some series.

11.8 Even Answers

⑧ Use Root Test

Converges only at center ($x=0$)

Radius of convergence, $R=0$

Interval of Convergence, $I = \{0\}$

↑
The set with only $x=0$

11.9 Even & Alternate Answers

$$\textcircled{4} f(x) = \frac{3}{1-x^4} = \sum_{n=0}^{\infty} 3x^{4n} \text{ for } -1 < x < 1$$

$$\textcircled{13} \textcircled{a} \sum_{n=1}^{\infty} (-1)^{n+1} n x^{n-1}, R=1$$

$$\textcircled{b} \sum_{n=2}^{\infty} \frac{1}{2} (-1)^n n(n-1) x^{n-2}$$

$$\textcircled{c} \sum_{n=2}^{\infty} \frac{1}{2} (-1)^n n(n-1) x^n$$

} Alternate Answers

$$\textcircled{14} \textcircled{a} f(x) = \ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \text{ w/ } R=1$$

$$\text{or} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

$$\textcircled{b} f(x) = x \ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{n+1} \text{ w/ } R=1$$

$$\text{or} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{n+1}}{n}$$

$$\text{or} = \sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n-1}$$

$$\textcircled{c} f(x) = \ln(x^2+1) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{n+1}}{n+1} \text{ w/ } R=1 \leftarrow x^{2n+2}$$

$$\text{or} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n}}{n}$$

$$\textcircled{15} \ln 5 - \sum_{n=0}^{\infty} \frac{1}{5^{n+1}(n+1)} x^{n+1}, R=5$$

$$\textcircled{17} \sum_{n=1}^{\infty} \frac{n}{2^{n+1}} x^{n+2}, R=2$$

} Alternate Answers

11.11 Even Answers

(2) $T_0(x) = 1$

(a) $T_1(x) = 1 - (x-1) = 2-x$

$$T_2(x) = 1 - (x-1) + (x-1)^2$$

$$T_3(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3$$

(b)

x	f(x) = 1/x	T ₀	T ₁	T ₂	T ₃
0.9	1.1̄	1	1.1	1.11	1.111
1.3	0.7692	1	0.7	0.79	0.763

(c) What do you notice about convergence and accuracy?

(14a) $T_2(x) = 1 - 2(x-1) + \frac{6}{2!}(x-1)^2 = 1 - 2(x-1) + 3(x-1)^2$

(16a) $T_4(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}(x-\frac{\pi}{6}) - \frac{1}{4}(x-\frac{\pi}{6})^2 - \frac{\sqrt{3}}{12}(x-\frac{\pi}{6})^3 + \frac{1}{48}(x-\frac{\pi}{6})^4$

(18a) $T_3(x) = \ln 3 + \frac{2}{3}(x-1) - \frac{4/9}{2!}(x-1)^2 + \frac{16/27}{3!}(x-1)^3$

(20a) $T_3(x) = (x-1) + \frac{1}{2}(x-1)^2 - \frac{1}{6}(x-1)^3$

11.10 Even Answers

$$\textcircled{64} \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \textcircled{66} e^{3/5} \quad \textcircled{68} e^{-\ln 2} = \frac{1}{2}$$