Students are expected to complete homework assignments on their own before referring to the following pages. The answers and hints are designed to check work and clarify problems. The original intent of the layout was for display in class after assignments had been completed. Students should use the following information as help to understand the exercises and master the concepts.

Calculus C

Chapter 10

Even Answers & Hints for Homework

10.1 Even Answers and More

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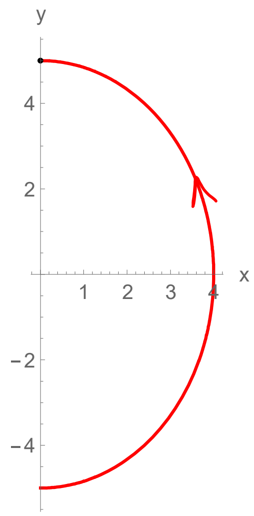

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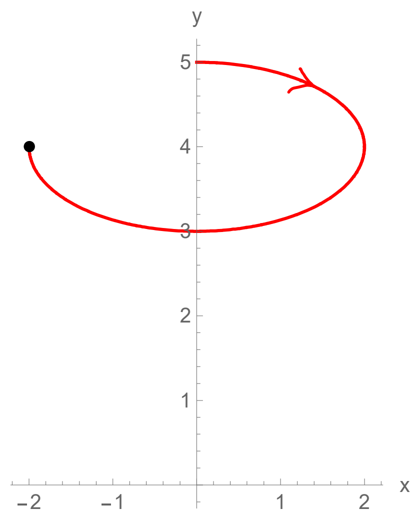

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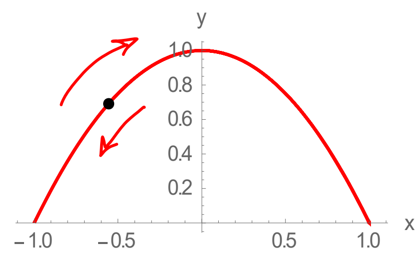

24 (a) III (b) I (c) IV (d) II

28 (a) V (b) I (c) II (d) VI (e) IV (f) III

33 a Note the book has a different answer than what I describe in notes. Either is fine.

Book Answer: $x = 2\cos t,\ y = 1 - 2\sin t,\ 0 \le t \le 2\pi$

This comes from replacing t with $-t$ in the standard (counterclockwise) parametrization.

Standard (Counterclockwise): $\begin{cases} x = 2\cos t \\ y = 2\sin t + 1 \end{cases} \Rightarrow$ Plugging in $-t$ (clockwise) $\Rightarrow \begin{cases} x = 2\cos(-t) \\ y = 2\sin(-t) + 1 \end{cases}$

Since $\cos(-t) = \cos t$ and $\sin(-t) = -\sin t$, this makes the answer $x = 2\cos t,\ y = -2\sin t + 1$

Alternate Answer: $x = 2\sin t,\ y = 2\cos t + 1,\ \frac{\pi}{2} \le t \le \frac{5\pi}{2}$

This comes from switching the cos and sin. However, to have it begin at the correct point, the domain changes too.

10.2 Answers with Setups

37) $\int_1^2 \sqrt{1+4t^2}\,dt$

38) $\int_{-3}^3 \sqrt{e^{2t}+4t^2}\,dt$

39) $\int_0^{2\pi} \sqrt{3-2\sin t-2\cos t}\,dt$

40) $\int_1^5 \sqrt{\frac{1}{t^2}+\frac{1}{4(t+1)}}\,dt = \int_1^5 \frac{t+2}{2t\sqrt{t+1}}\,dt$

41) $\int_0^1 6t\sqrt{1+t^2}\,dt = 2(2\sqrt{2}-1)$

43) $\int_0^2 \sqrt{\frac{1}{(1+t)^4}+\frac{1}{(1+t)^2}}\,dt = \int_0^2 \frac{\sqrt{t^2+2t+2}}{(1+t)^2}\,dt$

57) $S = 2\pi\int_0^1 (t^2+1)e^t\sqrt{e^{2t}(t+1)^2(t^2+2t+2)}\,dt$

$\uparrow (te^t+e^t)^2+((t^2+1)e^t+e^t(2t))^2$

58) $S = \int_0^{\pi/3} 2\pi \sin 3t\sqrt{\sin^2 2t+9\cos^2 3t}\,dt$

59) $S = \int_0^1 2\pi t^2\sqrt{9t^4+4t^2}\,dt$

60) $S = \int_0^1 2\pi\cdot 3t^2\sqrt{(3-3t^2)^2+(6t)^2}$

65) $S = \int_0^5 2\pi(3t^2)\,6t\sqrt{1+t^2}\,dt$

$\uparrow$ From $\sqrt{(6t)^2+(6t^2)^2}$

66) $S = \int_0^1 2\pi(e^t-t)\sqrt{(e^t-1)^2+(2e^{t/2})^2}\,dt$

$\hookrightarrow$ simplifies to $(e^t+1)^2$

Parametric Worksheet Answers

① $x = t^2 - 10,\ y = 10 - t^2$

$$\frac{dx}{dt} = 2t,\quad \frac{dy}{dt} = -2t \qquad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2t}{2t} = -1$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt} = \frac{\frac{d}{dt}[-1]}{dx/dt} = \frac{0}{2t} = 0$$

What does each mean in terms of the graph?
Write out your conclusions.

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② $x = t\cos t,\ y = t\sin t$

$$\frac{dx}{dt} = \cos t - t\sin t,\quad \frac{dy}{dt} = \sin t + t\cos t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t + t\cos t}{\cos t - t\sin t}$$

$$\left.\frac{dx}{dt}\right|_{t=\frac{7\pi}{3}} = \frac{1}{2} - \frac{7\pi}{3}\left(\frac{\sqrt{3}}{2}\right) \approx -5.8483$$

$$\left.\frac{dy}{dt}\right|_{t=\frac{7\pi}{3}} = \frac{\sqrt{3}}{2} + \frac{7\pi}{3}\left(\frac{1}{2}\right) \approx 4.53122$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{7\pi}{3}} = \frac{\frac{\sqrt{3}}{2} + \frac{7\pi}{6}}{\frac{1}{2} - \frac{7\pi\sqrt{3}}{6}} = \frac{3\sqrt{3} + 7\pi}{3 - 7\sqrt{3}\pi} \approx -0.7748$$

What does each mean in terms of the graph?
Write out your conclusions.

What do you know about $\frac{d^2y}{dx^2}$ at $t = \frac{7\pi}{3}$?
Is it positive, negative, zero, or undefined?

10.3 Even Answers

(28) (a) circle, radius 5, center (2,3)

Cartesian (Rectangular): $(x-2)^2+(y-3)^2=25$

Parametric: $x=5\cos t+2$, $y=5\sin t+3$

(b) circle, radius 4, center at origin

Polar: $r=4$

(56) (a) V (b) II (c) VI (d) III (e) I (f) IV

(58) $$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{2-\sqrt{3}}{1-2\sqrt{3}} = \frac{1-\sqrt{3}/2}{-\sqrt{3}+\frac{1}{2}}$$

(60) $$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{-1/2}{\sqrt{3}/6} = \frac{-3}{\sqrt{3}} = -\sqrt{3}$$

(62) $$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{1+-\sqrt{3}/2}{-\sqrt{3}-\sqrt{3}/2} = \frac{2-3}{-2\sqrt{3}-\sqrt{3}} = \frac{-1}{-3\sqrt{3}} = \frac{\sqrt{3}}{9}$$

Polar Worksheet Answers

① $r = 2$

$\left.\frac{dr}{d\theta}\right|_{\theta=0} = 0$ $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=0} = \infty$

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{4}} = 0$ $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = -1$

★ For each derivative consider the result graphically. Do this for each problem.

② $r = 2\cos\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=0} = 0$ $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=0} = \infty$

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{4}} = -\sqrt{2}$ $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = 0$

★ Be sure to also do Part III on separate paper. What did you learn from this?

③ $r = 2\sin 2\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=0} = 4$ $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=0} = 0$

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{4}} = 0$ $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = -1$

④ $r = 2\cos\theta + \frac{1}{2}$

$\left.\frac{dr}{d\theta}\right|_{\theta=0} \to$ zero $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=0} \to$ Undefined

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{3}} \to$ Negative $\quad$ $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} \to$ Positive

$\left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{3}} \to$ Negative $\quad$ $\left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{3}} \to$ Negative $\quad$ $\left.\frac{d^2y}{dx^2}\right|_{\theta=\frac{\pi}{3}} \to$ Negative

Do this type of analysis for #1, 2, and 3.