

If the focus is at the origin, or pole (as in the previous diagrams), we can use the eccentricity ( $e$ ) and the distance from the pole to the directrix ( $d$ ) to write the polar equation of the conic.

$$r = \frac{ed}{1 + e \cos \theta}, r = \frac{ed}{1 - e \cos \theta}, r = \frac{ed}{1 + e \sin \theta}, r = \frac{ed}{1 - e \sin \theta}$$

$\cos \theta \Rightarrow$  Vertical directrix (like  $r = \sec \theta$ )

$\sin \theta \Rightarrow$  Horizontal directrix (like  $r = \csc \theta$ )

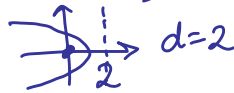
(+)  $\Rightarrow$  Directrix to right or above pole

(-)  $\Rightarrow$  Directrix to left or below pole

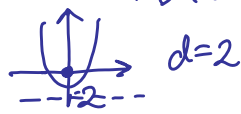
Examples: (Analyze first, calculator second.)

*It's a little sloppy, but I hope you get the idea.*

$r = \frac{2}{1 + \cos \theta}$   
 $e = 1 \Rightarrow$  parabola  
 $\cos \theta \Rightarrow$  vert.  
 $+$   $\Rightarrow$  right



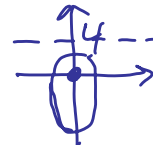
$r = \frac{2}{1 - \sin \theta}$   
 $e = 1 \Rightarrow$  parabola  
 $\sin \theta \Rightarrow$  horiz.  
 $-$   $\Rightarrow$  below



$r = \frac{2}{1 - 2 \cos \theta}$   
 $e = 2 \Rightarrow$  Hyperbola  
 $\cos \theta \Rightarrow$  vert.  
 $-$   $\Rightarrow$  left



$r = \frac{2}{1 + 0.5 \sin \theta}$   
 $e = 0.5 \Rightarrow$  Ellipse  
 $ed = 2 \Rightarrow 0.5d = 2 \Rightarrow d = 4$



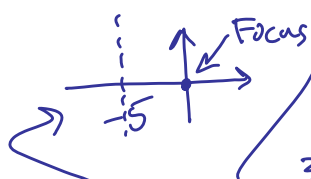
Example: For  $r = \frac{10}{3 - 2 \cos \theta}$  (a) find the eccentricity, (b) identify the conic, (c) give an equation for the directrix, and (d) sketch the conic. (Hint: First rewrite in  $\frac{ed}{1 - e \cos \theta}$  form.)

(c) Find the equation of this conic rotated counterclockwise about the origin by  $\frac{\pi}{4}$  radians.

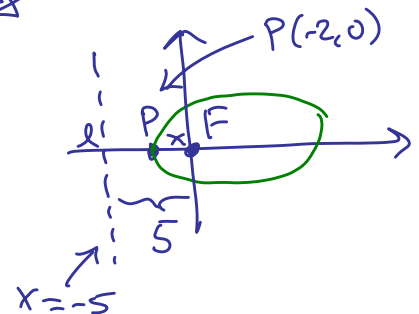
$$r = \frac{10}{3 - 2 \cos \theta} \cdot \frac{1/3}{1/3}$$

$$r = \frac{10/3}{1 - \frac{2}{3} \cos \theta}$$

$e = \frac{2}{3}$   $ed = 10/3$   $\cos \theta \Rightarrow$  vert.  
 $\frac{2}{3}d = \frac{10}{3}$   $(-) \Rightarrow$  Left  
 $d = 5$  directrix equation:  $x = -5$



To find vertex (P) use  $e = \frac{|PF|}{|PQ|}$   
 $\frac{2}{3} = \frac{x}{5-x}$   
 $10 - 2x = 3x$   
 $10 = 5x$   
 $2 = x$



- Reminders:
- In polar form, the primary focus is at the origin.
  - The directrix will never intersect the conic.