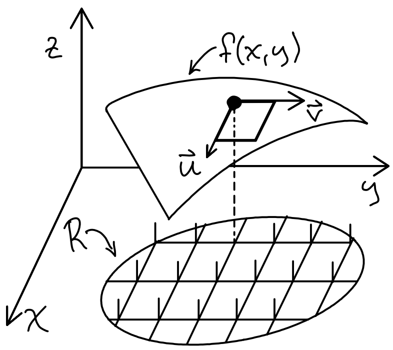
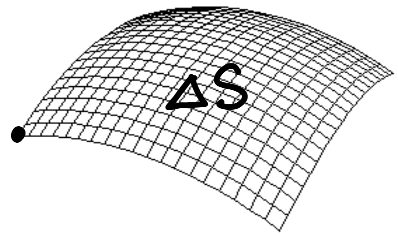
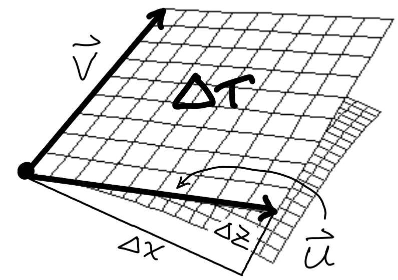
Surface Area (page 1 of 2)



Small piece of surface



Tangent plane approximation



$\Delta S \approx \Delta T$ where ΔT is the tangent plane at (x_i, y_j, z_{ij}) ← Point closest to the origin over region R_{ij}

Area of tangent plane $= \|\vec{u} \times \vec{v}\|$

$$\vec{u} = \Delta x_i \hat{\imath} + \Delta z_i \hat{k} = \Delta x_i \hat{\imath} + 0\hat{\jmath} + \frac{\Delta z_i}{\Delta x_i}\Delta x_i \hat{k}$$
$$\vec{v} = \Delta y_j \hat{\jmath} + \Delta z_j \hat{k} = 0\hat{\imath} + \Delta y_j \hat{\jmath} + \frac{\Delta z_j}{\Delta y_j}\Delta y_j \hat{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{\left(\frac{\Delta z_i}{\Delta x_i}\right)^2 (\Delta x_i \Delta y_j)^2 + \left(\frac{\Delta z_j}{\Delta y_j}\right)^2 (\Delta x_i \Delta y_j)^2 + (\Delta x_i \Delta y_j)^2} \quad \text{Let } \Delta x_i \Delta y_j = \Delta A_{ij}$$

$$\text{Surface Area} = S = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \sqrt{\left(\frac{\Delta z_i}{\Delta x_i}\right)^2 + \left(\frac{\Delta z_j}{\Delta y_j}\right)^2 + 1}\, \Delta A_{ij}$$

$$S = \iint_R \sqrt{1 + (f_x(x,y))^2 + (f_y(x,y))^2}\, dA = \iint_S dS$$

<u>Single Variable</u>

Length of Interval $= \int_a^b dx$

Arc Length

$= s = \int_a^b ds = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

<u>Multivariable</u>

Area of region $= \iint_R dA$

Surface Area

$= S = \iint_S dS = \iint_R \sqrt{1 + (f_x)^2 + (f_y)^2}\, dA$

Example: Find the area of the surface given by $z = f(x,y) = 2 + \frac{2}{3}x^{3/2}$ over the region $R = \{(x,y): 0 \le x \le 1, 0 \le y \le 1 - x\}$

Example: Find the area of the portion of the paraboloid $z = 16 - x^2 - y^2$ in the first octant.