Surfaces in Space (page 1 of 3)

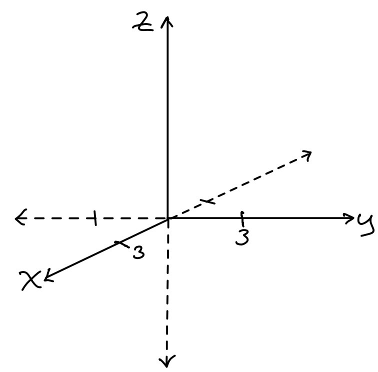
Spheres: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$
Planes: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Cylinders

Given a generating curve C in a plane and line L not in a parallel plane, the set of all lines parallel to L and intersecting C is called a cylinder. The parallel lines are called rulings. For our purposes, we will only use curves C that lie in one of the coordinate planes. Also, unless stated otherwise, we will only use $L \perp C$. In these cases, the equation of the cylinder is the equation of its generating curve.

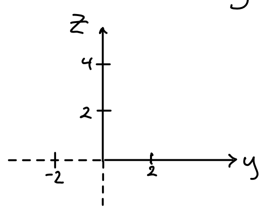
In other words, a cylinder is the 3-dimensional extension of a 2-dimensional graph. The graph extends in the direction of the missing variable. The graph takes on all possible values of whatever is not specified in the equation—much like $y=4$ is a point in 1 dimension, a line in 2 dimensions, and a plane in 3 dimensions.

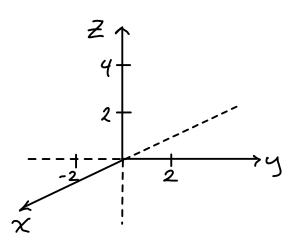
Ex: C is the curve $x^2+y^2=9$.
Let L be the line
$x=3,\ y=0,\ z=t$.



But not all cylinders are "cylinders."
Ex: $y^2+z=4$

2D →





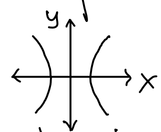
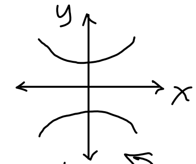
3D

Quadric Surfaces : 2nd degree equations in 3-space have the following form:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

Recall: Conic sections in 2-space

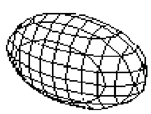
parabola: $y = ax^2 + b$, ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

hyperbolas: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ 

opens about axis of positive variable

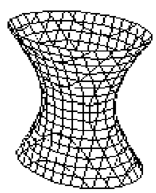
$(x-h)$ and $(y-k)$ shifts center of conic from $(0,0)$ to (h,k).

xy-terms cause rotation.



Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ What happens if $a=b=c$?

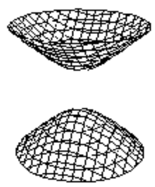
ellipse in all three planes



Hyperboloid of One Sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

hyperbola in 2 planes, ellipse in other

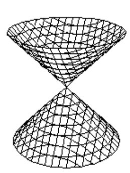
Note: 1 hyperbola opens about x-axis and other about y-axis



Hyperboloid of Two Sheets: $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

hyperbola in 2 planes, ellipse in other

Note: Both hyperbolas open about z-axis



Elliptic Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$

lines in 2 planes, ellipse in other

Elliptic Paraboloid: $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

parabolas in 2 planes, ellipse in other

Hyperbolic Paraboloid: $z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$

parabolas in 2 planes, hyperbola in other

$(x-h)$ and $(y-k)$ shift center to (h,k)

Example: Identify the surface and its center.

$$4x^2+y^2-4z^2-16x-6y-16z+9=0$$

Sketching Practice: Identify and sketch.

① $x^2+\frac{y^2}{4}+z^2=1$

② $x^2-y+z^2=0$

③ $z^2=x^2+\frac{y^2}{4}$

④ $\frac{x^2}{16}-\frac{y^2}{16}+z=0$