Abigail Brown
www.abbymath.com

POWER SERIES (page 1 of 2)

$\sum_{n=0}^{4} \frac{1}{n^2+1} = 1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17}$ is an example of a <u>series</u>.

$\sum_{n=0}^{\infty} \frac{1}{n^2+1} = 1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26} + \cdots + \frac{1}{n^2+1} + \cdots$ is an <u>infinite series</u>.

$\sum_{n=0}^{4} \frac{1}{n^2+1} x^n = 1 + \frac{1}{2}x + \frac{1}{5}x^2 + \frac{1}{10}x^3 + \frac{1}{17}x^4$ is a <u>polynomial</u>.

$\sum_{n=0}^{\infty} \frac{1}{n^2+1} x^n = 1 + \frac{1}{2}x + \frac{1}{5}x^2 + \frac{1}{10}x^3 + \cdots + \frac{1}{n^2+1}x^n + \cdots$ is a <u>power series</u>.

If x is a variable, then a power series centered at a has the form:

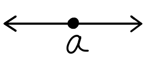
$$\sum_{n=0}^{\infty} C_n (x-a)^n = C_0 + C_1(x-a) + C_2(x-a)^2 + \cdots + C_n(x-a)^n + \cdots$$

Note: If $a=0$, $\sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots + C_n x^n + \cdots$

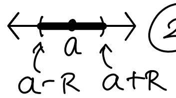
Note: We say $(x-a)^0 = 1$ even if $x=a$.

The primary question when working with a power series $f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n$ is "For what values of x does the series converge?" (I.e., what is the domain of f?) This set of x-values is the <u>interval of convergence</u>.

For the power series $\sum_{n=0}^{\infty} C_n(x-a)^n$ only one of the following is true:



① The series converges only at $x=a$.



② $\exists R > 0$ such that the series converges absolutely for $|x-a| < R$ and diverges for $|x-a| > R$.

③ The series converges absolutely for all x.

R = radius of convergence

Example: $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$ Find the interval of convergence.

Use Ratio Test: We want $\lim_{n\to\infty} \left| \frac{(2x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(2x)^n} \right| < 1$ for convergence.

$= \lim_{n\to\infty} |$

$\therefore R =$ ______ and the interval of convergence is ____________.

② $\sum_{n=0}^{\infty} \frac{(2n)!\,x^n}{n!}$

③ $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n\,3^n}$

Abigail Brown
www.abbymath.com

$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n3^n}$ has $R=3$ and interval of convergence $-1 < x < 5$.
But we also need to check the endpoints.

If $x=-1$, $\sum_{n=0}^{\infty} \frac{(-1-2)^n}{n3^n} = \sum_{n=0}^{\infty} \frac{(-3)^n}{n3^n}$ which __________ by __________.

If $x=5$, $\sum_{n=0}^{\infty} \frac{(5-2)^n}{n3^n} = \sum_{n=0}^{\infty} \frac{3^n}{n3^n}$ which __________ by __________.

$\therefore$ The interval of convergence is __________.

Try: $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$

Properties

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$$

$$f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1} = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \cdots$$

Note: starts at $n=1$

$$\int f(x)\,dx = C + \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1} = C + c_0(x-a) + c_1\frac{(x-a)^2}{2} + c_2\frac{(x-a)^3}{3} + \cdots$$

The radius of convergence is the same for each.
The interval of convergence may differ at the endpoints.