RATIO AND ROOT TESTS (page 1 of 2)

ST **The Ratio Test** For $\sum a_n$ with $a_n \neq 0$, let $\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$.

The series $\sum a_n$
1. Converges absolutely if $\rho < 1$
2. Diverges if $\rho > 1$ or is infinite

The test is inconclusive if $\rho = 1$. (Proof uses direct comparison test.)

Ex: $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$

Ex: $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

Ex: $\sum_{n=1}^{\infty} n^{-3}$

Ex: $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Ex: $\sum_{n=1}^{\infty} \frac{1}{n^p}$

ST **The Root Test** For $\sum a_n$, let $\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$.

The series $\sum a_n$
1. Converges absolutely if $\rho < 1$
2. Diverges if $\rho > 1$ or is infinite.

The test is inconclusive if $\rho = 1$.

This works well for series with n^{th} powers.

Ex: $\sum_{n=1}^{\infty} \left(\frac{-2}{3n+1}\right)^{3n}$

Note: If it's easier, write $\rho = \lim_{n\to\infty} |a_n|^{1/n}$

☆ Guidelines for Testing Series for Convergence or Divergence ☆

1. n^{th} Term Test
2. Special Types: geometric*, p-series, telescoping*, alternating.
3. Integral, Ratio, Root Tests
4. Comparisons: Direct or Limit

*Has a formula for the sum.

There are 10 tests to consider.

More than one method may work for some series.
Always indicate which method you are using.

Try: Indicate method and decide converges/diverges if done easily.

1. $\sum_{n=1}^{\infty} \frac{1}{6n+7}$
2. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$
3. $\sum_{n=1}^{\infty} \frac{3n}{4n+5}$
4. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$
5. $\sum_{n=1}^{\infty} (-1)^n n^{-4/5}$
6. $\sum_{n=1}^{\infty} \left(\frac{2n^2+1}{5n^2-n-2}\right)^{2n}$
7. $\sum_{n=1}^{\infty} \frac{n!}{10!}$
8. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
9. $\sum_{n=1}^{\infty} 4$