INTRODUCTION TO
MATHEMATICA AND LINEAR ALGEBRA

Basic Input

Let's create the matrix $A = \begin{bmatrix} -1 & 4 & 1 \\ 1 & 9 & -2 \\ 6 & 4 & -8 \end{bmatrix}$ There are several ways to do this.

- The fastest way to type in a matrix is to make it a list of lists using braces. Each sub-list is a row.
  A={{-1, 4, 1},{1,9,-2},{6,4,-8}} Note that you do not need spaces.
  If you want to see it looking like a matrix, type **A//MatrixForm** and press **Shift-Enter**.

- Another way is to go to the "Insert" menu and choose "Table/Matrix... > New..." <u>*Make sure "Matrix" is selected*</u> and type in 3 Rows and 3 Columns. Click "OK"
- Click on the cell in position 1,1 and type "-1." Use **Tab** to move from one cell to the next.
- Set **A=** to the matrix (in front of it) and press **Shift-Enter** to evaluate it.

Let's get the template for matrix $B = \begin{bmatrix} 1 & -2 & 5 \\ 4 & -5 & 8 \\ -3 & 3 & -3 \end{bmatrix}$ a different way.

- Type **B=** and go to the "Palettes" menu and select "ClassroomAssistant." The palette window should appear on the right. Select "Typesetting." Select the palette button for creating a 2×2 matrix.
- In the main notebook window, place the cursor inside the matrix template. Press **Ctrl-Enter** to add a row and press **Ctrl-,** (comma) to add a column. Type the entries for *B* as you did *A* using Tab.
- Don't forget to press **Shift-Enter** to evaluate it.

Matrix Operations

Multiplication

- Add *A* and *B* by typing **A+B**
- Multiply *A* and *B* by typing **A B** using a *space* between them.
- Check your answer by doing the multiplication yourself.
- Now try multiplying *A* and *B* by typing **A.B** using a *period* between them. What is the difference?
- What is the difference between **B^2** and **MatrixPower[B,2]**?

Inverses

- Let's augment matrix *A* with *I*.
- Type: **Join[A, IdentityMatrix[3], 2]** and evaluate.
- Oops! Let's name it. Go back and type **F=** in front of "Join...." and re-evaluate.
- Type **MatrixForm[F]** to see it better. (Note: Be careful not to define a matrix in Matrix Form. This can cause syntax errors.)
- Now, type **RowReduce[F]//MatrixForm** to reduce it and see it as a matrix.
- Type **Ainv=Take[RowReduce[F], {1,3},{4,6}]** to define Ainv from rows 1-3 and columns 4-6.
- Now multiply *Ainv* and *A* by typing **Ainv.A**
- What do you get? Does this make sense?
- Of course, this was the long method, but I hope you learned something along the way.
- You can also just type **Inverse[A]** and evaluate that.
- Find B^{-1}.

(Continued on next page.)

Other Operations

In the following be sure to use proper matrix multiplication using the . (period) between matrices.

Try the following:

1. Find the trace of *A* and of *B* using **Tr[***name of matrix***]**.
2. Find the transpose of *B* using **Transpose[***name of matrix***]**.
3. Use *Mathematica* to verify that $(AB)^T = B^T A^T \neq A^T B^T$ for matrices *A* and *B* as defined above by calculating the value of all three parts $(AB)^T$, $B^T A^T$, and $A^T B^T$ separately.

Since *B* is singular, we need to change it to create a matrix that has an inverse.
Let's change b_{32} from 3 to 0.
Type: **B = ReplacePart[B, 0, {3, 2}]**.

4. Use *Mathematica* to verify that $(AB)^{-1} = B^{-1} A^{-1} \neq A^{-1} B^{-1}$ for matrices *A* and *B*.
5. Use *Mathematica* to verify that $(B^{-1})^T = (B^T)^{-1}$ for the matrix *B*.

To calculate a determinant use **Det[A]**.

6. Find det(*A*) and det(*B*)
7. Find det(*AB*) and det(*A* + *B*).
8. Find $\det(A^T)$, $\det(B^T)$, $\det(A^{-1})$, $\det(B^{-1})$.
9. Make *B* the same as it was originally: **B = ReplacePart[B, 3, {3, 2}]** Find det(*B*).
10. What conclusions do you draw from these examples? Write your thoughts in the space below.

Exercises

You may find the following exercises in the book interesting to try. These are optional.

Chapter 1 Supplementary Exercises (p.74):
1, 2, 8, 15, 16, 27, 9, 10, 12ab (Read and think about 12c), 19, 22