

Quiz #4 (5.1 - 5.6) **Part I**

Scientific calculator allowed.

_____ 30 pts.

This section is "open notebook." Clearly show *all* work.

1) Do problem 20a in section 5.3.
(3 points)

2) Define *row space*. (2 points)

Quiz #4 (5.1 - 5.6) **Part II**

Scientific calculator allowed. *This section is “closed notebook.”*

Clearly show *all* work. Simplify and circle your answers.

- 1) List the missing axioms (by formula *or* description) that must hold for a set V , with elements \mathbf{u} and \mathbf{v} , to be a vector space. (1 point each - order does *not* matter.)
- a) Closure under vector addition
 - b) Additive Inverse
 - c) Additive Identity
 - d) Distributive Axiom - Vector over scalar addition
 - e) Distributive Axiom - Scalar over vector addition
 - f) _____
 - g) _____
 - h) _____
 - i) _____
 - j) _____

- 2) Let U be the set of all vectors with the form $\mathbf{u} = (-u, u, -u)$. Prove that U is a subspace of \mathbb{R}^3 . (2 points)

- 3) $S = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$
 Prove that S is a linearly independent set. (2 points)

4) Define *basis*. (2 points)

5) Prove that the set S from #3
 $S = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$
is a basis for \mathbb{R}^3 . (2 points)

6) $\mathbf{v}_1 = \langle -1, 1 \rangle$ and $\mathbf{v}_2 = \langle 6, 3 \rangle$ form a basis
for \mathbb{R}^2 .

Find the coordinate vector of $\langle 6, 0 \rangle$
relative to the basis above. (2 points)

$$\text{Let } A = \begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{bmatrix}$$

Note that the reduced row-echelon form of A is

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

7) Find a basis for the column space of A .
(2 points)

8) Find a basis for the nullspace of A .
(2 points)

9) Check that your results make sense by comparing the rank, nullity, and size of A . (2 point)

10) Let A be an $n \times n$ matrix with $\det(A) = 0$. True or False? (1 point each)

a) $\text{nullity}(A) = 0$ _____

b) The row vectors of A form a linearly dependent set _____

c) The row vectors of A span \mathbb{R}^n . _____

d) A is nonsingular _____