Tangent Plane and Normal Line Examples

1) Find and graph the tangent plane and normal line for
$f(x, y) = -x\,y\,e^{-x^2-y^2}$ at the point $\left(-\frac{1}{2},\ \frac{1}{2},\ \frac{1}{4\sqrt{e}}\right)$.

Plot the surface:

In[1]:=
```
Plot3D[-x * y * E^(-x^2 - y^2), {x, -1, 1}, {y, -1, 1}, PlotStyle → Blue, Mesh → None]
```

Change domain to center at point of interest and name this to be "surface" *for later use.*

In[2]:=
```
surface = Plot3D[-x * y * E^(-x^2 - y^2), {x, -1, 0}, {y, 0, 1}, PlotStyle → Blue, Mesh → None]
```

Find the derivative of F(x, y, z) = z - f(x, y) with respect to x:

In[3]:=
```
D[z + x * y * E^(-x^2 - y^2), x]
```

Find the derivative of F(x, y, z) with respect to x and substitute in values for x and y.

In[4]:=
```
D[z + x * y * E^(-x^2 - y^2), x] /. {x → -1/2, y → 1/2}
```

Do the same thing for the y and z partial derivatives.

In[5]:=
```
D[z + x * y * E^(-x^2 - y^2), y] /. {x → -1/2, y → 1/2}
D[z + x * y * E^(-x^2 - y^2), z] /. {x → -1/2, y → 1/2}
```

There is also a built-in gradient function: Grad.
You have to indicate the variables as the second argument to Grad. See below.

In[7]:=
```
Grad[z + x * y * E^(-x^2 - y^2), {x, y, z}]
```

In[8]:=
```
Grad[z + x * y * E^(-x^2 - y^2), {x, y, z}] /. {x → -1/2, y → 1/2, z -> 1/(4 √e)}
```

Make Mathematica solve a(x - x0) + b(y - y0) + c(z - z0) = 0 for z so it's easy to plot.
Note the double equals == within the equation.

In[9]:=
```
Solve[1/(4 √e) (x + 1/2) - 1/(4 √e) (y - 1/2) + 1 (z - 1/(4 √e)) == 0, z]
```

Graph the plane:

In[10]:=
```
plane = Plot3D[(-x + y)/(4 √e), {x, -3/4, -1/4}, {y, 1/4, 3/4}, Mesh → None]
```

Show the surface and the plane together:

In[11]:=
```
Show[surface, plane, BoxRatios → Automatic, PlotRange → All]
```

Graph the normal line:

In[12]:=
```
NormalLine =
  ParametricPlot3D[{1/(4 √e) t - 1/2, -1/(4 √e) t + 1/2, 1 t + 1/(4 √e)}, {t, 0, 0.3}, PlotStyle → Red]
```

Show all together:

In[13]:=
```
Show[surface, plane, NormalLine, BoxRatios → Automatic, PlotRange → All]
```

Extra: Plot the contour map for f(x, y) over the smaller and larger domains.

In[14]:=
```
ContourPlot[-x * y * E^(-x^2 - y^2), {x, -1, 0}, {y, 0, 1}]
```

In[15]:=
```
ContourPlot[-x * y * E^(-x^2 - y^2), {x, -1, 1}, {y, -1, 1}]
```

2) Find and graph the tangent plane and normal line for

$f(x, y) = 1 - \frac{1}{10}\left(x^2 + 4 y^2\right)$ at the point $\left(1, 1, \frac{1}{2}\right)$.

3) Find the line tangent to the curve of intersection of

$z = \sqrt{x^2 + y^2}$ and $2 x + y + 2 z = 20$ at $(3, 4, 5)$. Graph everything.