

# Surfaces Summary

Name: \_\_\_\_\_  
For use after Chapter 12 - [www.abbymath.com](http://www.abbymath.com)

Here are some thoughts to help you organize the information we have about different types of objects.

## • Lines and Planes (well, technically, lines are not surfaces)

Lines:  $x = at + x_0$   
 $y = bt + y_0$   
 $z = ct + z_0$   $\leftrightarrow$  Parametric form  $\vec{r}(t) = (at + x_0)\hat{i} + (bt + y_0)\hat{j} + (ct + z_0)\hat{k}$   
 Parallel vector =  $\langle a, b, c \rangle$

$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} = t$   $\leftarrow$  Symmetric Form

Planes:  $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$  Normal vector =  $\langle a, b, c \rangle$   
 $ax + by + cz + d = 0$

## • Cylinders: Equations that look 2D but live in 3D.

Generating curve exists in one coordinate plane and surface is created by dragging it in the direction of the free variable.

Eg.,  $y = x^2$ ,  $x = \sin z$ ,  $z = e^y$

## • Quadric Surfaces: $\pm \frac{x^2}{a^2} \pm \frac{y^2}{b^2} \pm \frac{z^2}{c^2} = 1$ or $0$ or one variable is linear

### ① Check Powers

All Three are Squared

Ellipsoid, Hyperboloids, Cone

### ② Check = 1 or 0

③ Check signs.  $\begin{matrix} \swarrow & \nearrow \\ \text{Cone} & \text{Ellipsoid} \end{matrix}$  (if not +++)

+++ Ellipsoid  
 +- - Hyp. of 1 sheet  
 + - - Hyp. of 2 sheets

Two Squared + One Linear

Paraboloids

### ② Isolate linear variable

### ③ Check signs of the quadratic variables

+ + or - - Elliptic Paraboloid (or circular)  
 + - or - + Hyperbolic Paraboloid

## • Simple Rectangular, Cylindrical, and Spherical Surfaces

$\hookrightarrow$  plane x3  
 $x \ y \ z$

$\hookrightarrow$  cylinder, plane, plane  
 $r \ \theta \ z$

$\hookrightarrow$  sphere,  $\frac{1}{2}$  plane,  $\frac{1}{2}$  cone  
 $\rho \ \theta \ \phi$