Integration Summary

Scalar Functions

interval length $= \int_a^b dx$ $\quad A = \int_a^b f\,dx$ $\quad$ arc length $= s = \int_C ds$ $\quad$ "curtain" area or mass $= \int_C f\,ds$

$A = \iint_R dA$ $\quad V = \iint_R f\,dA$ $\quad$ surface area $= S = \iint_S dS$ $\quad$ mass of surface lamina $= \iint_S f\,dS$

$V = \iiint_E dV$ $\quad$ mass $= \iiint_E f\,dV$ $\quad$ Note: Integral represents "mass" if *f* is a density function.

Vector Functions

work $= \int_C \mathbf{F}\cdot d\mathbf{r}$

$= \int_C \mathbf{F}\cdot \mathbf{T}\,ds$

$= \int_a^b \mathbf{F}\cdot \mathbf{r}'(t)\,dt$

$= \int_C P\,dx + Q\,dy + R\,dz \leftarrow$ differential form

flux $= \iint_S \mathbf{F}\cdot d\mathbf{S}$

$= \iint_S \mathbf{F}\cdot \mathbf{N}\,dS$

$= \iint_R \mathbf{F}\cdot \nabla G\,dA$

$= \iint_D \mathbf{F}\cdot (\mathbf{r}_u \times \mathbf{r}_v)\,dA \leftarrow$ parametric form

$$\mathbf{T} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

$$\mathbf{N} = \frac{\nabla G}{\|\nabla G\|}$$

Elements of Integration

$dA = dy\,dx,\ r\,dr\,d\theta,\ du\,dv$
$dV = dz\,dy\,dx,\ r\,dz\,dr\,d\theta,\ \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

$d\mathbf{r} = \mathbf{T}\,ds = \mathbf{r}'(t)\,dt$
$d\mathbf{S} = \mathbf{N}\,dS = \nabla G\,dA$

$$ds = \|\mathbf{r}'(t)\|\,dt = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\ dt$$

$dS = \|\nabla G\|\,dA = \sqrt{[g_x(x,y)]^2 + [g_y(x,y)]^2 + 1}\ dA$ where $z = g(x,y)$ and $G(x,y,z) = z - g(x,y)$

$= \|\mathbf{r}_u \times \mathbf{r}_v\|\,dA$ where S is given by $\mathbf{r}(u,v) \leftarrow$ parametric form

C \ **F**	**Conservative** (∃ a potential function *f* such that **F**=∇*f*)	**Not Conservative**	
Closed	$\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ Note: Green's, Stokes's, and Fundamental Theorem also apply in this case.	Green's Theorem (2D) $\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA$	Stokes's Theorem (3D) $\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \mathbf{curl}\,\mathbf{F}\cdot d\mathbf{S}$
Not Closed	Fundamental Theorem of Line Integrals $\int_C \mathbf{F}\cdot d\mathbf{r} = f(x(b),y(b),z(b)) - f(x(a),y(a),z(a))$ where $\mathbf{F} = \nabla f$	$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}\cdot \mathbf{r}'(t)\,dt$ Complete the line integral	

If S is *closed*: Divergence Theorem $\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \operatorname{div}\mathbf{F}\,dV$

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