Chapter 16 Integrals

Abby Brown 5/15/04

Integral \ Integrand	SCALAR: 1	SCALAR: Scalar function f	VECTOR: vector field $\vec{F}$	
Line Integrals	arc length $= \int_C ds$	mass of curve $= \int_C f\,ds$	Work* $= \int_C \vec{F} \cdot d\vec{r}$	$\vec{r}(t)$ describes curve C
	$ds = \lVert \vec{r}\,'(t) \rVert\, dt$		$d\vec{r} = \vec{r}\,'(t)\,dt$	
Surface Integrals	surface area $= \iint_S dS$	mass of surface $= \iint_S f\,dS$	flux $= \iint_S \vec{F} \cdot d\vec{S}$	$G(x,y,z)$ describes surface S
	$dS = \lVert \vec{\nabla} G \rVert\, dA$		$d\vec{S} = \vec{\nabla} G\, dA$	

If S is $\vec{r}(u,v)$, use $\vec{r}_u \times \vec{r}_v$ instead of $\vec{\nabla} G$.

f = density if finding mass

$G(x,y,z) = z - g(x,y)$
$G(x,y,z) = y - g(x,z)$
$G(x,y,z) = x - g(y,z)$

*Work $= \int_C \vec{F} \cdot dr$

C \ $\vec{F}$	Conservative	Not Conservative	
Closed	$\int_C \vec{F} \cdot dr = 0$	$\int_C \vec{F} \cdot d\vec{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$	Green's Theorem (2D)
Not Closed	$\int_C \vec{F} \cdot d\vec{r} = f\vert_{t=b} - f\vert_{t=a}$	$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F} \cdot \vec{r}\,'(t)\,dt$	Long Way No Shortcuts
	Fundamental Theorem of Line Integrals $\vec{\nabla} f = \vec{F}$		

Stokes's Thm.: $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\,\vec{F} \cdot d\vec{S}$ (3D)

Divergence Thm.: $\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \text{div}\,\vec{F}\, dV$ (Closed S)