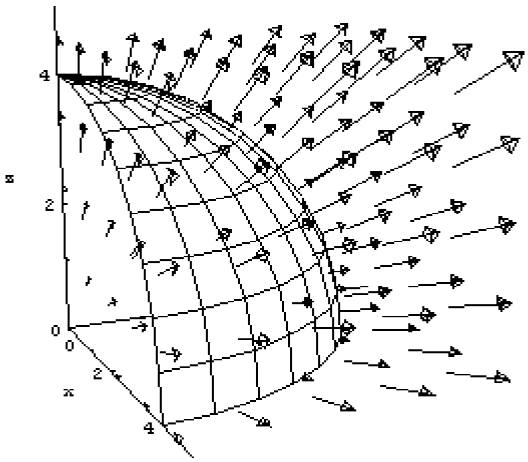
A flux integral adds up across S all of the components of $\vec{F}$ normal to S.

Example: Find the flux of $\vec{F}$ through S, $\iint_S \vec{F}\cdot d\vec{S}$ for
$\vec{F}(x,y,z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $S: x^2+y^2+z^2=16$ (first octant)
(Let $\vec{N}$ be the upward unit normal.)

$\vec{F} = \langle x, y, z \rangle$



$$\iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}\cdot\vec{\nabla}G\,dA$$ ← surface

So we need to find $\vec{\nabla}G$.

Surface: $x^2+y^2+z^2=16$

$$z = \sqrt{16-x^2-y^2}$$

$$z = g(x,y) = \sqrt{16-x^2-y^2}$$

$$G(x,y,z) = z - g(x,y)$$

$$G = z - \sqrt{16-x^2-y^2}$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}\cdot\vec{\nabla}G\,dA \qquad \Rightarrow \vec{\nabla}G = \left\langle \frac{x}{\sqrt{16-x^2-y^2}}, \frac{y}{\sqrt{16-x^2-y^2}}, 1 \right\rangle$$

$$= \iint_D \langle x, y, z\rangle \cdot \left\langle \frac{x}{\sqrt{16-x^2-y^2}}, \frac{y}{\sqrt{16-x^2-y^2}}, 1 \right\rangle dA$$

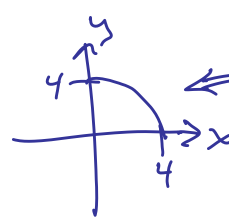
$$= \iint_D \frac{x^2}{\sqrt{16-x^2-y^2}} + \frac{y^2}{\sqrt{16-x^2-y^2}} + z\,dA$$

Problem: x, y, and z. We need to get only x & y since we used $z=g(x,y)$. Use $z=\sqrt{16-x^2-y^2}$ to replace z.

$$= \iint_D \frac{x^2+y^2}{\sqrt{16-x^2-y^2}} + \sqrt{16-x^2-y^2}\,dA$$

Common denominator

$$= \iint_D \frac{x^2+y^2}{\sqrt{16-x^2-y^2}} + \frac{16-x^2-y^2}{\sqrt{16-x^2-y^2}}\,dA$$



Polar

$$= \iint_D \frac{16}{\sqrt{16-x^2-y^2}}\,dA$$

$$= \iint_D \frac{16}{\sqrt{16-r^2}}\,r\,dr\,d\theta \xrightarrow{\text{u-substitution}} = 32\pi$$