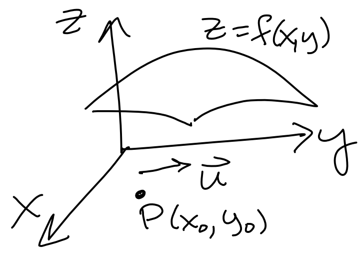
Section 14.6 – Summary Notes



For a 3D surface $z=f(x,y)$, a <u>partial derivative</u> gives the slope in a direction parallel to one of the coordinate axes. A <u>directional derivative</u> gives the slope of the surface in any direction we want. For example,

$$D_{\vec{u}}f(x_0,y_0)=\vec{\nabla}f(x_0,y_0)\cdot\vec{u}$$

is the derivative of f at $P(x_0,y_0)$ in the direction of <u>unit vector</u> $\vec{u}$. Each point has infinitely many directional derivatives since there are infinitely many directions in which to take the derivative.

Question: In which direction is the <u>maximum</u> derivative?

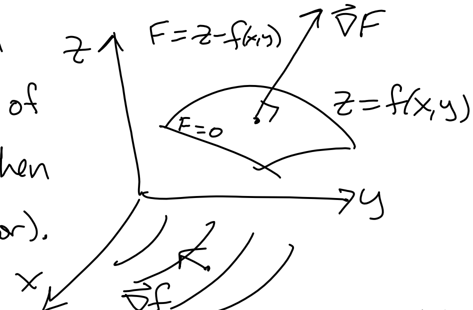
Answer: The gradient $\vec{\nabla}f$. (This <u>vector</u> is in the xy-plane)

Question: What is the maximum derivative?

Answer: The magnitude of the gradient $\|\vec{\nabla}f\|$ (a <u>scalar</u>).

For a 4D situation $w=f(x,y,z)$, such as temperature, we can also find partial and directional derivatives. $D_{\vec{u}}f(x,y,z)=\vec{\nabla}f\cdot\vec{u}$ is the rate of change of f in the direction of $\vec{u}$ (a 3D unit vector). The direction of maximum derivative is still $\vec{\nabla}f$ and the value of the maximum derivative is still $\|\vec{\nabla}f\|$.

For a surface $z=f(x,y)$, $\vec{\nabla}f$ is orthogonal to the <u>level curves</u> in the xy-plane. For a 4D situation $w=f(x,y,z)$, $\vec{\nabla}f$ is orthogonal to the <u>level surfaces</u> in xyz-space. If we want to find a vector orthogonal to a <u>surface</u> $z=f(x,y)$, we can convert $z=f(x,y)$ into the level surface $(F=0)$ of a new 4D situation $F(x,y,z)=z-f(x,y)$ and then take the gradient of this: $\vec{\nabla}F(x,y,z)$ (A 3D vector). Use $\vec{\nabla}F$ to find tangent planes, normal lines, etc.



Abby Brown 3/19/07
www.abbymath.com