

Chapter 15 - Multiple Integrals

Each assignment has a total possible of **10 points**. For each section, self-grade for completion. (You may use $\frac{1}{2}$ points.) I trust that you will give an honest evaluation of your own work. Your signature at the bottom indicates that this is an honest, accurate assessment of your work. Grades will be verified, as explained in class. Try additional problems for extra practice. Each assignment lists "Priority Problems" with a "PP" designation. Full credit awarded for completion of full assignment. *Assignments are subject to change. Any changes will be announced in class.*

_____ 15.2: p.964 #1 – 17 odd
 15.3: p.972 #1, 3, 5, 39 – 50
 PP: 15.3 #39 – 50

_____ 15.1: Read p. 950 – 959
 15.2: p.964 #25 – 35 odd, Optional: 33, 34, 37 w/ *Mathematica*
 15.3: p.972 #7 – 27 odd
 PP: 15.3 #19 – 27 odd

_____ 15.4: p.978 #1 – 6, 7 – 35 odd (Skip 33)
 15.5: Read the section
 PP: 15.4 #19 – 31 odd, 35

Quiz 15.1 – 15.4
Optional checkpoint and/or review.
Does not need to be included with HW.

_____ 15.6: p.998 #1 – 21 odd, 12, 22, 27, 28, 29 – 35 odd*
 *At least 2 different ways with different inside (first) variable, do more for extra practice.
 PP: #12, 15, 17, 21, 29, 31, 33

_____ Stewart 4th ed. 15.6 Surface Area (Handout attached): p.1008 #1 – 21 odd (skip 13), 4, 6
 Hint #11: The cylinder creates the boundary $r = a \cos \theta$ in the xy -plane. It's easier if you convert to polar after finding the integrand. Set up only okay.
 For 15 & 17 Set up only. For 21 use "polar" in xz -plane.
 PP: #3, 4, 6, 7, 21

_____ 15.7: p.1004 Do any of #1 – 14 to review (optional)
 Assignment: #15, 16, 17, 19, 21, 23a, 27, 28

15.8: p. 1010 Do any of #1 – 16 to review (optional)
 Assignment: #17 – 21, 23, 25, 39, 40

15.9: Look over section. Note result of Example 4. Do p. 1020 #1 – 4 (optional)
 PP: 15.7 #19, 21, 27, 28, 15.8# 23, 25, 39, 40

_____ **Total (60 Points)**

Signature: _____ Date: _____

Verified By: _____

15.6 Stewart 4th ed. Surface Area

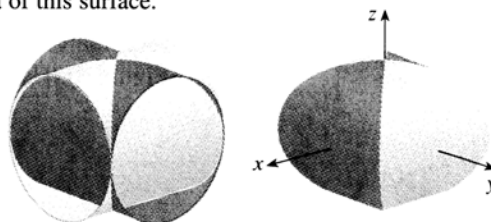
Name: _____

Copy exercises and show all work on separate paper.

1–12 □ Find the area of the surface.

1. The part of the plane $z = 2 + 3x + 4y$ that lies above the rectangle $[0, 5] \times [1, 4]$
2. The part of the plane $2x + 5y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 9$
3. The part of the plane $3x + 2y + z = 6$ that lies in the first octant
4. The part of the surface $z = x + y^2$ that lies above the triangle with vertices $(0, 0)$, $(1, 1)$, and $(0, 1)$
5. The part of the cylinder $y^2 + z^2 = 9$ that lies above the rectangle with vertices $(0, 0)$, $(4, 0)$, $(0, 2)$, and $(4, 2)$
6. The part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy -plane
7. The part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$
8. The surface $z = \frac{2}{3}(x^{3/2} + y^{3/2})$, $0 \leq x \leq 1$, $0 \leq y \leq 1$
9. The part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$
10. The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$
11. The part of the sphere $x^2 + y^2 + z^2 = a^2$ that lies within the cylinder $x^2 + y^2 = ax$ and above the xy -plane
12. The part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$
13. (a) Use the Midpoint Rule for double integrals (see Section 15.1) with four squares to estimate the surface area of the portion of the paraboloid $z = x^2 + y^2$ that lies above the square $[0, 1] \times [0, 1]$.
(b) Use a computer algebra system to approximate the surface area in part (a) to four decimal places. Compare with the answer to part (a).
14. (a) Use the Midpoint Rule for double integrals with $m = n = 2$ to estimate the area of the surface $z = xy + x^2 + y^2$, $0 \leq x \leq 2$, $0 \leq y \leq 2$.

- (b) Use a computer algebra system to approximate the surface area in part (a) to four decimal places. Compare with the answer to part (a).
15. Find the exact area of the surface $z = x^2 + 2y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$.
16. Find the exact area of the surface $z = 1 + x + y + x^2$, $-2 \leq x \leq 1$, $-1 \leq y \leq 1$.
Illustrate by graphing the surface.
17. Find, to four decimal places, the area of the part of the surface $z = 1 + x^2y^2$ that lies above the disk $x^2 + y^2 \leq 1$.
18. Find, to four decimal places, the area of the part of the surface $z = (1 + x^2)/(1 + y^2)$ that lies above the square $|x| + |y| \leq 1$. Illustrate by graphing this part of the surface.
19. Show that the area of the part of the plane $z = ax + by + c$ that projects onto a region D in the xy -plane with area $A(D)$ is $\sqrt{a^2 + b^2 + 1} A(D)$.
20. If you attempt to use Formula 2 to find the area of the top half of the sphere $x^2 + y^2 + z^2 = a^2$, you have a slight problem because the double integral is improper. In fact, the integrand has an infinite discontinuity at every point of the boundary circle $x^2 + y^2 = a^2$. However, the integral can be computed as the limit of the integral over the disk $x^2 + y^2 \leq t^2$ as $t \rightarrow a^-$. Use this method to show that the area of a sphere of radius a is $4\pi a^2$.
21. Find the area of the finite part of the paraboloid $y = x^2 + z^2$ cut off by the plane $y = 25$. [Hint: Project the surface onto the xz -plane.]
22. The figure shows the surface created when the cylinder $y^2 + z^2 = 1$ intersects the cylinder $x^2 + z^2 = 1$. Find the area of this surface.



Answers to Odd-Numbered Exercises

1. $15\sqrt{26}$ 3. $3\sqrt{14}$ 5. $12 \sin^{-1} \frac{3}{5}$ 7. $(\pi/6)(17\sqrt{17} - 5\sqrt{5})$ 9. $(2\pi/3)(2\sqrt{2} - 1)$ 11. $a^2(\pi - 2)$ 13. (a) ≈ 1.83 (b) ≈ 1.8616 15. $\frac{2}{3} + \frac{8}{5} \ln 5$ 17. 3.3213 21. $(\pi/6)(101\sqrt{101} - 1)$