

**Quiz 14.1 - 14.5**

Show all work and circle answers.  
No Calculators. 3 points each.

15 pts.

Name: Key  
Per.: \_\_\_\_\_

1) Where is the function

$$f(x, y) = \frac{\sqrt{x}}{x - y^2} \text{ continuous?}$$

All points  $(x, y)$  such that  
 $x \geq 0$  and  $x \neq y^2$

OR

All points except when  $x < 0$  or  
 $x = y^2$

2) Evaluate  $\frac{\partial}{\partial y} \left[ \frac{x^4 - y}{x^2 + y} \right]$ .

$$\frac{d}{dy} [(x^4 - y)(x^2 + y)^{-1}] \rightarrow \text{product rule}$$

$$= (-1)(x^2 + y)^{-1} + (x^4 - y)(-1)(x^2 + y)^{-2}(1)$$

$$= \frac{-1}{x^2 + y} - \frac{x^4 - y}{(x^2 + y)^2}$$

$$= \frac{-1(x^2 + y) - (x^4 - y)}{(x^2 + y)^2} \leftarrow \text{Quotient Rule also works}$$

$$= \frac{-x^2 - y - x^4 + y}{(x^2 + y)^2} = \frac{-x^2 - x^4}{(x^2 + y)^2}$$

3) Find  $\frac{\partial^2}{\partial y \partial x} \left[ \frac{4x^2}{y} + \frac{y^2}{2x} \right]$ . Be sure to

differentiate in the correct order.

$$\text{Let } f(x, y) = \frac{4x^2}{y} + \frac{y^2}{2x}$$

$$\frac{df}{dx} = \frac{8x}{y} - \frac{y^2}{2x^2}$$

$$\frac{\partial^2 f}{\partial y \partial x} = -\frac{8x}{y^2} - \frac{2y}{2x^2}$$

$$= \frac{-8x}{y^2} - \frac{y}{x^2}$$

check that  
 $x$  derivative  
is first,  
then  $y$   
Important

4) Let  $w(x, y, z) = xy^2 + xz^2$ ,  $x(s, t) = t + 1$ ,  
 $y(s, t) = t - 1$ , and  $z(s, t) = st$ .  
Using the appropriate chain rule, find

(a)  $\frac{\partial w}{\partial t}$  in general & (b)  $\frac{\partial w}{\partial t}$  when  $t = 0$ ,  $s = 1$ . *all d, Not d*

$$\textcircled{a} \frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt}$$

$$\begin{aligned} \frac{\partial w}{\partial t} &= (y^2 + z^2)(1) + (2xy)(1) + (2xz)(s) \\ &= (t-1)^2 + (st)^2 + 2(t+1)(t-1) + 2(t+1)(st)s \\ &= t^2 - 2t + 1 + s^2t^2 + 2t^2 - 2 + 2s^2t^2 + 2s^2t \\ &= 3t^2 - 2t - 1 + 3s^2t^2 + 2s^2t \end{aligned}$$

$$\textcircled{b} \frac{\partial w}{\partial t} \Big|_{\substack{t=0 \\ s=1}} = 3(0)^2 - 2(0) - 1 + 3(1)^2(0)^2 + 2(1)^2(0)$$

$$= -1$$

Indicate when pluggin in values.  
This will be checked for notation on tests.

5) Write the symbolic expression for the definition of the first partial derivative of  $f(x, y)$  with respect to  $x$ .

$$f_x(x, y) = \frac{\partial f}{\partial x}(x, y)$$

$$= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

Bonus) Explain what a partial derivative of  $f$  means in terms of the graph of  $f$ . (+2 pts.)

A partial derivative is the slope of the graph in the direction parallel to the positive axis for the variable that you are taking the partial derivative w/respect to.

# Contour Quiz

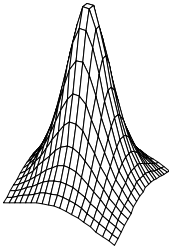
\_\_\_\_\_   
 5 pts.

Name: Key

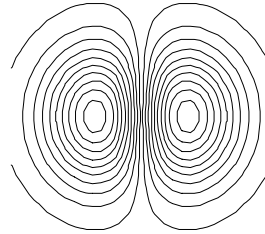
Per.: \_\_\_\_\_

Write the letter of the level curve graph that matches the three-dimensional surface. (1 point each.)

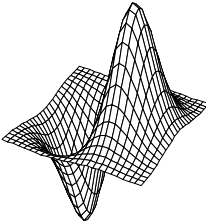
C 1.



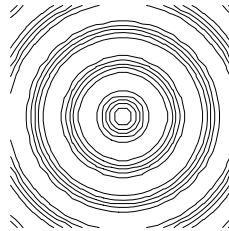
A.



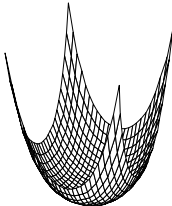
A 2.



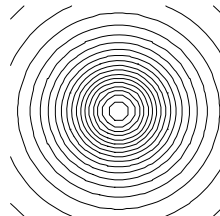
B.



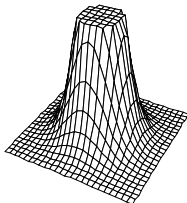
D 3.



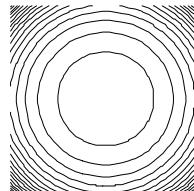
C.



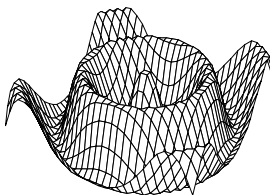
E 4.



D.



B 5.



E.

