Students are expected to complete homework assignments on their own before referring to the following pages. The answers and hints are designed to check work and clarify problems. The original intent of the layout was for display in class after assignments had been completed. Students should use the following information as help to understand the exercises and master the concepts.

Calculus D

Chapter 16

Even Answers & Hints for Homework

16.1 & 16.5 Answers

16.1

(11) II (12) IV (13) I (14) III

(15) IV (16) I (17) III (18) II

(22) $3\sec^2(3x-4y)\,\hat{\imath} - 4\sec^2(3x-4y)\,\hat{\jmath}$

(24) $\left\langle \cos\frac{y}{z},\ -\frac{x}{z}\sin\frac{y}{z},\ \frac{xy}{z^2}\sin\frac{y}{z} \right\rangle$

(26) $\frac{x}{\sqrt{x^2+y^2}}\,\hat{\imath} + \frac{y}{\sqrt{x^2+y^2}}\,\hat{\jmath}$

(29) III (30) IV (31) II (32) I

(30) $\hookrightarrow \langle 2x+y, x\rangle$

(32) $\hookrightarrow \frac{\cos\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}(x\hat{\imath}+y\hat{\jmath})$

16.5

(12) Hint: $\vec{F}$ is a vector field, f is a scalar-valued function.
(more details in class)

(14) Not conservative
$\text{curl}\,\vec{F} = \langle 2x^2yz - 2x^2yz,\ -2xy^2z - 2xyz,\ 2xyz^2 - xz^2\rangle$

(16) $f(x,y,z) = xe^z + y + K$

(18) $f(x,y,z) = \sin xy + \cos z + K$

(22)
$$\begin{aligned}\text{div}\,\vec{F} &= \frac{\partial}{\partial x}[f(y,z)] + \frac{\partial}{\partial y}[g(x,z)] + \frac{\partial}{\partial z}[h(x,y)] \\ &= 0 + 0 + 0 \\ &= 0\end{aligned}$$

16.2 Even Answers

40 $\frac{1}{2}(15 + \cos 1 - \cos 4)$

16.3 - Even Answers

2) 6

28) Show $\overrightarrow{\text{curl}}\,\vec{F} \neq \vec{0}$ and $\therefore$ not conservative
(Use curl even though it is 16.5)

16.4 Even Answers

(2) Both ways $= 9/2$

(6) $30(1-\cos 2)$

(8) 0 (10) 0 (12) -16 (14) $-\pi$

16.6 Even Answers and Hints

3-6 Eliminate parameters, convert to x,y,z & identify

29,30 Graphing optional

4 $\frac{x^2}{4}+\frac{y^2}{9}=1, \ 0 \le z \le 2$

6 $y = x^2 + z^2$

34 $x + y - 2z = 0$

36 $y = 0$

38 $9\sqrt{30}\,\pi$ (Note: Solve in rectangular, like Chapter 15)

40 $\sqrt{107}$

42 $\frac{1}{24}(26^{3/2} - 10^{3/2})$ (Note: Solve in rectangular, like Chapter 15)

47 Hint: Inside root is a square trinomial

ex: $\sqrt{a^2+2ab+b^2} = \sqrt{(a+b)^2} = (a+b)$

then integrate.

16.7 Surface Integrals HW Setups

I took these from the solution guide. It is possible that some may have alternate setups.

Note: You will need u-sub for some like this ↗

$$\int x^3\sqrt{1+x^2}\,dx \qquad u = 1+x^2 \;\rightarrow x^2 = u-1$$
$$= \int x\cdot x^2(1+x^2)^{1/2}\,dx \qquad du = 2x\,dx \qquad \text{etc.}$$

Be sure to also read the hints on the assignment sheet.

(5) $\int_0^3\int_0^2 x^2y(1+2x+3y)\sqrt{14}\,dy\,dx$

(7) $\int_0^1\int_0^{1-x} y(1-x-y)\sqrt{3}\,dy\,dx$

(9) $\int_0^{\pi/2}\int_0^1 (u\sin v)(u\cos v)\,u\sqrt{1+4u^2}\,du\,dv$

(11) $\iint_R x^2(x^2+y^2)\sqrt{2}\,dA = \sqrt{2}\int_0^{2\pi}\int_1^3 (r\cos\theta)^2(r^2)\,r\,dr\,d\theta$

(13) $\iint_R (x^2+z^2)\sqrt{1+4(x^2+z^2)}\,\underset{dx\,dz}{dA} = \int_0^{2\pi}\int_0^2 r^2\sqrt{1+4r^2}\,r\,dr\,d\theta$

(15) $\iint_R (x^2+y^2)2\,dA = \int_0^{2\pi}\int_0^2 r^2\cdot 2\cdot r\,dr\,d\theta$

(17) $\vec{r}(u,v) = u\hat{\imath} + \cos v\,\hat{\jmath} + \sin v\,\hat{k}$

$$\int_0^{\pi/2}\int_0^3 (\sin v + u^2\cos v)\,1\,du\,dv$$

(19) $\int_0^1\int_0^1 2x^2y + 2y^2(4-x^2-y^2) + x(4-x^2-y^2)\,dy\,dx$

(21) $\int_0^1\int_0^{1-x} -(1-x-y)\,dy\,dx$ (note: downward orientation)

(23) $\iint_R \dfrac{-x^2}{\sqrt{4-x^2-y^2}}\,dA = \int_0^{\pi/2}\int_0^2 -(r\cos\theta)^2(4-r^2)^{-1/2}\,r\,dr\,d\theta$

(27) Do like first example in Divergence Thm. notes.

16.9 Even Answers

(26) $\vec{F}(x,y,z) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}\vec{F}\, dV$$

$$\operatorname{div}\vec{F} = 1 + 1 + 1 = 3$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E 3\, dV = 3\iiint_E dV = 3(\text{volume of } E)$$

$$\therefore\ \underset{\uparrow\text{volume}}{V(E)} = \frac{1}{3}\iint_S \vec{F}\cdot d\vec{S}$$

16.8 Even Answers and Hints

Pay attention to the direction of the theorem

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S \text{curl}\,\vec{F}\cdot d\vec{S}$$

"Find $\int_C \vec{F}\cdot d\vec{r}$ using Stokes' Thm." $\Rightarrow$ Do $\iint_S \text{curl}\,\vec{F}\cdot d\vec{S}$

"Find $\iint_S \text{curl}\,\vec{F}\cdot d\vec{S}$ using Stokes' Thm." $\Rightarrow$ Do $\int_C \vec{F}\cdot d\vec{r}$

2) -18π

8) $2e-4$ (needs integration by parts)

10) 9π (plane is the surface, cylinder creates region boundary)

12a) π (hyperbolic paraboloid is surface, cylinder creates region boundary)

13) Solve both ways. Show both sides of theorem have the same answer. Hint: $\vec{r}(t) = \langle \cos t, \sin t, 1\rangle$, $0 \le t \le 2\pi$ Why?
Answer: π