Calculus C Review

This worksheet includes many topics, but **NOT** everything needed for the final exam.

Do all exercises by hand first on separate paper. (* # *) Indicates problem numbers. You don't need to put those in code.

Integration

Evaluate the integrals below. Which method would you use to complete each if you were to solve it by hand?

Note: After evaluating each in *Mathematica,* evaluate **Expand[%]** to un-simplify the result to compare to your work.

```
(* 1 *) Integrate[x^2 Cos[3 x], x]
```

For number 1, for example, this means $\int x^2 \cos(3\,x)\,dx$.

```
(* 2 *) Integrate[x^2 / Sqrt[1 - x^2], x]
(* 3 *) Integrate[(Cos[t])^2 (Tan[t])^2, t]
(* 4 *) Integrate[1 / (x^2 + 5 x + 6), x]
```

Add limits of integration to the integral above (as shown below) and evaluate. Follow the examples to explore an improper integral.

```
(* 5 *) Integrate[1 / (x^2 + 5 x + 6), {x, 0, 1}]
Plot[1 / (x^2 + 5 x + 6), {x, 0, 1000}]
NIntegrate[1 / (x^2 + 5 x + 6), {x, 0, 1}]  (* Note the change to NIntegrate *)
NIntegrate[1 / (x^2 + 5 x + 6), {x, 0, 100}]
Manipulate[NIntegrate[1 / (x^2 + 5 x + 6), {x, 0, b}], {b, 0, 1000}]
(* 6 *) Integrate[1 / (x^2 + 5 x + 6), {x, 0, Infinity}] (* Note change back to Integrate *)
```

Use N[%] to get a decimal approximation to the answer. Or evaluate N[.... fill in previous answer ...] with your answer above.

Infinite Series

For each series below, predict convergence or divergence. What test would you use? Use **NSum** for decimal approximations.

```
(* 7 *) Sum[1 / n^4, {n, 1, Infinity}]
Manipulate[NSum[1 / n^4, {n, 1, nmax}], {nmax, 1, 10, 1}]
ListPlot[Table[NSum[1 / n^4, {n, 1, nmax}], {nmax, 1, 10}]]
(* 8 *) Sum[5 * (4 / 3) ^n, {n, 0, Infinity}]
(* 9 *) Sum[3^n * n^2 / n!, {n, 1, Infinity}]
(* 10 *) Sum[(-1) ^n / (n^3 - 1), {n, 2, Infinity}]  (* also try NSum *)
(* 11 *) Sum[n^2 * E^(-n), {n, 2, Infinity}]
```

Below is a brief review of power series and Taylor series. **(* 12 *)** Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n\,2^n}$.

```
Plot[Table[Sum[(x + 2) ^n / (n * 2^n), {n, 1, nmax}], {nmax, 1, 10}], {x, -5, 5}, PlotRange → 20]
```

(* 13 *) Find a power series representation for $f(x) = \frac{1}{x^2+4}$ using geometric series and write out the first three terms.

(* 14 *) Then find a second-degree polynomial approximation for $f(x)$ using Maclaurin series (i.e., find $T_2(x)$).

```
Normal[Series[1 / (x^2 + 4), {x, 0, 8}]]
polys = Table[Normal[Series[1 / (x^2 + 4), {x, 0, n}]], {n, 1, 10}]
plot1 = Plot[polys, {x, -3, 3}, PlotRange → 1 / 2]
plot2 = Plot[1 / (x^2 + 4), {x, -3, 3}, PlotRange → 1 / 2, PlotStyle → {Black, Thick}]
Show[plot1, plot2]
```

Introduction to Differential Equations

Example: $y' = 2x - y,\ y(0) = 0.$ **(* 15 *)** Use Euler's method with step size 1 to approximate $y(3)$. **(* 16 *)** Then find the exact solution $y(x)$ and use that solution to find the exact value for $y(3)$.

```
(* Fill in with your points or calculate with the next bit of code. *)
EulerList = {{0, 0}, {__, __}, {__, __}, {__, __}}

(* Skip this if you just want to use your own calculated points as above. *)
h = 1; ptlist = {{0, 0}};
EulerList = Last@Table[xn = Last[ptlist][[1]]; yn = Last[ptlist][[2]];
   ptlist = Append[ptlist, {xn + h, yn + h * (2 xn - yn)}], {n, 1, 3, h}]

Euler = ListPlot[EulerList, Joined → True, PlotStyle → Red]

slopes = VectorPlot[Normalize[{1, 2 x - y}], {x, -4, 4}, {y, -4, 4},
  VectorStyle → "Segment", VectorColorFunction → None, Axes → True, Frame → False]

Show[slopes, Euler]

(* Fill in your exact solution or use the next bit of code to calculate and graph. *)
soln = Plot[___ ___ ___ ___ ___, {x, -3, 3}, PlotStyle → Purple]

(* Skip this if you just want to use your own calculated points as above. *)
soln = Plot[Evaluate[y[x] /. DSolve[{y'[x] == 2 x - y[x], y[0] == 0}, y, x]], {x, -3, 3}, PlotStyle → Purple]

Show[slopes, Euler, soln, PlotRange → 4]

(* Fill in your solution here to get decimal answer. Do not use //N for the exact answer. *)
(___ ___ ___ ___ ___) /. x → 3 // N
```

Parametric and Polar Equations

(* 17 *) Find the equation of the line tangent to $x = t^2 + 1$, $y = t - t^3$ at $t = 1$.

```
plot1 = ParametricPlot[{1 + t^2, t - t^3}, {t, -2, 2}, AxesOrigin → {0, 0}]

(* Fill in with your answer. Just the LEFT (mx+b) side of the y=mx+b answer. *)
plot2 = Plot[___ ___ ___, {x, 0, 3}, PlotStyle → Red]

Show[plot1, plot2, AxesOrigin → {0, 0}]

(* Try this if you have time. *)
m[t_] = D[t - t^3, t] / D[1 + t^2, t];
Manipulate[Show[ParametricPlot[{1 + t^2, t - t^3}, {t, -2, 2}],
  Plot[m[t0] (x - (1 + t0^2)) + (t0 - t0^3), {x, 0, 5}, PlotStyle → Red],
  AxesOrigin → {0, 0}, PlotRange → {{0, 5}, {-3, 3}}],
 {t0, -2, 2}]
```

(* 18 *) Find the area enclosed by one petal of r = cos(4 θ).

```
PolarPlot[Cos[4 t], {t, 0, 2 Pi}]

Integrate[___ ___ ___, {t, __, __}]

(* Try this if you have time. *)
r[t_] = Cos[4 t];
Manipulate[Show[PolarPlot[r[t], {t, 0, 2 Pi}], PolarPlot[r[t], {t, 0, t0}, PlotStyle → {Red, Thick}],
  Graphics[{PointSize[0.02], Point[{r[t0] * Cos[t0], r[t0] * Sin[t0]}]}]],
 {t0, 0.001, 2 Pi}]
```