

Larson 7.7 L'Hôpital's Rule

Name: _____

Copy exercises and show work on separate paper.

In Exercises 1–6, evaluate the limit (a) using techniques from Chapters 1 and 3 and (b) using L'Hôpital's Rule.

1. $\lim_{x \rightarrow 3} \frac{2(x-3)}{x^2-9}$
2. $\lim_{x \rightarrow -1} \frac{2x^2-x-3}{x+1}$
3. $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3}$
4. $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x}$
5. $\lim_{x \rightarrow \infty} \frac{5x^2-3x+1}{3x^2-5}$
6. $\lim_{x \rightarrow \infty} \frac{2x+1}{4x^2+x}$

In Exercises 7–26, evaluate the limit, using L'Hôpital's Rule if necessary. (In Exercise 13, n is a positive integer.)

7. $\lim_{x \rightarrow 2} \frac{x^2-x-2}{x-2}$
8. $\lim_{x \rightarrow -1} \frac{x^2-x-2}{x+1}$
9. $\lim_{x \rightarrow 0} \frac{\sqrt{4-x^2}-2}{x}$
10. $\lim_{x \rightarrow 2} \frac{\sqrt{4-x^2}}{x-2}$
11. $\lim_{x \rightarrow 0} \frac{e^x-(1-x)}{x}$
12. $\lim_{x \rightarrow 0^+} \frac{e^x-(1+x)}{x^3}$
13. $\lim_{x \rightarrow 0^+} \frac{e^x-(1+x)}{x^n}$
14. $\lim_{x \rightarrow 1} \frac{\ln x}{x^2-1}$
15. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$
16. $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$
17. $\lim_{x \rightarrow 0} \frac{\arcsin x}{x}$
18. $\lim_{x \rightarrow 1} \frac{\arctan x - (\pi/4)}{x-1}$
19. $\lim_{x \rightarrow \infty} \frac{3x^2-2x+1}{2x^2+3}$
20. $\lim_{x \rightarrow \infty} \frac{x-1}{x^2+2x+3}$
21. $\lim_{x \rightarrow \infty} \frac{x^2+2x+3}{x-1}$
22. $\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$
23. $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}}$
24. $\lim_{x \rightarrow \infty} \frac{\sin x}{x - \pi}$
25. $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$
26. $\lim_{x \rightarrow \infty} \frac{e^x}{x}$

Answers to Odd-Numbered Exercises

49.	x	$\frac{x}{(\ln x)^4}$	2.811	4.498	0.720	0.036	0.001	0.000
	10^{10}	10^8	10^6	10^4	10^2	10^0	10^{-8}	10^{-10}

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In Exercises 27–40, describe the type of indeterminate form (if any) that is obtained with direct substitution. Then evaluate the limit, using L'Hôpital's Rule when necessary. (For a geometric approach to Exercise 27, see the article by John H. Mathews in the May, 1992 issue of *The College Mathematics Journal*.)

27. $\lim_{x \rightarrow 0^+} (-x \ln x)$
28. $\lim_{x \rightarrow 0^+} x^2 \cot x$
29. $\lim_{x \rightarrow \infty} \left(x \sin \frac{1}{x} \right)$
30. $\lim_{x \rightarrow \infty} x \tan \frac{1}{x}$
31. $\lim_{x \rightarrow 0^+} x^{1/x}$
32. $\lim_{x \rightarrow 0^+} (e^x + x)^{1/x}$
33. $\lim_{x \rightarrow \infty} x^{1/x}$
34. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x$
35. $\lim_{x \rightarrow 0^+} (1+x)^{1/x}$
36. $\lim_{x \rightarrow \infty} (1+x)^{1/x}$
37. $\lim_{x \rightarrow 2^+} \left(\frac{8}{x^2-4} - \frac{x}{x-2} \right)$
38. $\lim_{x \rightarrow 2^+} \left(\frac{1}{x^2-4} - \frac{\sqrt{x-1}}{x^2-4} \right)$
39. $\lim_{x \rightarrow 1^+} \left(\frac{3}{\ln x} - \frac{2}{x-1} \right)$
40. $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{x^2} \right)$

In Exercises 43–48, use L'Hôpital's Rule to determine the comparative rates of increase of the functions

$$f(x) = x^m, \quad g(x) = e^{nx}, \quad \text{and} \quad h(x) = (\ln x)^n$$

where $0 < n$, $0 < m$, and $x \rightarrow \infty$. The limits obtained in these exercises suggest that $(\ln x)^n$ approaches infinity more slowly than x^m , which, in turn, approaches infinity more slowly than e^{nx} .

43. $\lim_{x \rightarrow \infty} \frac{x^2}{e^{5x}}$
44. $\lim_{x \rightarrow \infty} \frac{x^3}{e^{2x}}$
45. $\lim_{x \rightarrow \infty} \frac{(\ln x)^3}{x}$
46. $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x^3}$
47. $\lim_{x \rightarrow \infty} \frac{(\ln x)^n}{x^m}$
48. $\lim_{x \rightarrow \infty} \frac{x^m}{e^{nx}}$

49. Complete the table to show that x eventually “overpowers” $(\ln x)^4$.

x	10	10^2	10^4	10^6	10^8	10^{10}
$\frac{(\ln x)^4}{x}$						

50. Complete the table to show that e^x eventually “overpowers” x^5 .

x	1	5	10	20	30	40	50	100
$\frac{e^x}{x^5}$								