Larson 7.7 L'Hôpital's Rule

Name: ______________________

Copy exercises and show all work on separate paper.

In Exercises 1–6, evaluate the limit (a) using techniques from Chapters 1 and 3 and (b) using L'Hôpital's Rule.

1. $\lim_{x\to 3}\frac{2(x-3)}{x^2-9}$
2. $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}$
3. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$
4. $\lim_{x\to 0}\frac{\sin 4x}{2x}$
5. $\lim_{x\to\infty}\frac{5x^2-3x+1}{3x^2-5}$
6. $\lim_{x\to\infty}\frac{2x+1}{4x^2+x}$

In Exercises 7–26, evaluate the limit, using L'Hôpital's Rule if necessary. (In Exercise 13, *n* is a positive integer.)

7. $\lim_{x\to 2}\frac{x^2-x-2}{x-2}$
8. $\lim_{x\to -1}\frac{x^2-x-2}{x+1}$
9. $\lim_{x\to 0}\frac{\sqrt{4-x^2}-2}{x}$
10. $\lim_{x\to 2^-}\frac{\sqrt{4-x^2}}{x-2}$
11. $\lim_{x\to 0}\frac{e^x-(1-x)}{x}$
12. $\lim_{x\to 0^+}\frac{e^x-(1+x)}{x^3}$
13. $\lim_{x\to 0^+}\frac{e^x-(1+x)}{x^n}$
14. $\lim_{x\to 1}\frac{\ln x}{x^2-1}$
15. $\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}$
16. $\lim_{x\to 0}\frac{\sin ax}{\sin bx}$
17. $\lim_{x\to 0}\frac{\arcsin x}{x}$
18. $\lim_{x\to 1}\frac{\arctan x-(\pi/4)}{x-1}$
19. $\lim_{x\to\infty}\frac{3x^2-2x+1}{2x^2+3}$
20. $\lim_{x\to\infty}\frac{x-1}{x^2+2x+3}$
21. $\lim_{x\to\infty}\frac{x^2+2x+3}{x-1}$
22. $\lim_{x\to\infty}\frac{x^2}{e^x}$
23. $\lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}}$
24. $\lim_{x\to\infty}\frac{\sin x}{x-\pi}$
25. $\lim_{x\to\infty}\frac{\ln x}{x}$
26. $\lim_{x\to\infty}\frac{e^x}{x}$

In Exercises 27–40, describe the type of indeterminate form (if any) that is obtained with direct substitution. Then evaluate the limit, using L'Hôpital's Rule when necessary. (For a geometric approach to Exercise 27, see the article by John H. Mathews in the May, 1992 issue of *The College Mathematics Journal.*)

27. $\lim_{x\to 0^+}(-x\ln x)$
28. $\lim_{x\to 0^+}x^2\cot x$
29. $\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right)$
30. $\lim_{x\to\infty}x\tan\frac{1}{x}$
31. $\lim_{x\to 0^+}x^{1/x}$
32. $\lim_{x\to 0^+}(e^x+x)^{1/x}$
33. $\lim_{x\to\infty}x^{1/x}$
34. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$
35. $\lim_{x\to 0^+}(1+x)^{1/x}$
36. $\lim_{x\to\infty}(1+x)^{1/x}$
37. $\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$
38. $\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right)$
39. $\lim_{x\to 1^+}\left(\frac{3}{\ln x}-\frac{2}{x-1}\right)$
40. $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)$

In Exercises 43–48, use L'Hôpital's Rule to determine the comparative rates of increase of the functions

$$f(x)=x^m,\quad g(x)=e^{nx},\quad \text{and}\quad h(x)=(\ln x)^n$$

where $0<n$, $0<m$, and $x\to\infty$. The limits obtained in these exercises suggest that $(\ln x)^n$ approaches infinity more slowly than x^m, which, in turn, approaches infinity more slowly than e^{nx}.

43. $\lim_{x\to\infty}\frac{x^2}{e^{5x}}$
44. $\lim_{x\to\infty}\frac{x^3}{e^{2x}}$
45. $\lim_{x\to\infty}\frac{(\ln x)^3}{x}$
46. $\lim_{x\to\infty}\frac{(\ln x)^2}{x^3}$
47. $\lim_{x\to\infty}\frac{(\ln x)^n}{x^m}$
48. $\lim_{x\to\infty}\frac{x^m}{e^{nx}}$

49. Complete the table to show that x eventually "overpowers" $(\ln x)^4$.

x	10	10^2	10^4	10^6	10^8	10^{10}
$\frac{(\ln x)^4}{x}$						

50. Complete the table to show that e^x eventually "overpowers" x^5.

x	1	5	10	20	30	40	50	100
$\frac{e^x}{x^5}$								

Answers to Odd-Numbered Exercises

1. $\frac{1}{3}$ **3.** $\frac{1}{4}$ **5.** $\frac{5}{3}$ **7.** 3 **9.** 0 **11.** 2

13. $n=1$: 0; $n=2$: $\frac{1}{2}$; $n\ge 3$: ∞ **15.** $\frac{2}{3}$ **17.** 1 **19.** $\frac{3}{2}$ **21.** ∞ **23.** 1

25. 0 **27.** $0\cdot\infty$, 0 **29.** $0\cdot\infty$, 1 **31.** Not indeterminate, 0

33. ∞^0, 1 **35.** 1^∞, e **37.** $\infty-\infty$, $-\frac{3}{2}$ **39.** $\infty-\infty$, ∞

41. a. $f(x)=x^2-25$, $g(x)=x-5$

b. $f(x)=(x-5)^2$, $g(x)=x^2-25$

c. $f(x)=x^2-25$, $g(x)=(x-5)^3$

43. 0 **45.** 0 **47.** 0

49.

x	10	10^2	10^4	10^6	10^8	10^{10}
$\frac{(\ln x)^4}{x}$	2.811	4.498	0.720	0.036	0.001	0.000