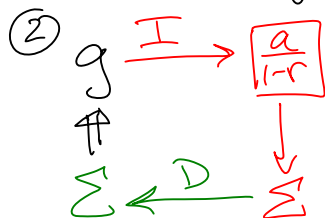


Last Example in Notes: Representations of Functions as Power Series

Ex: $f(x) = \frac{x^2}{(1-2x)^2}$ Note: $f(x) = x^2 \left(\frac{1}{(1-2x)^2} \right)$

Plan: ① $f \rightarrow x^2 g$ \leftarrow Let's deal with g , then put x^2 back in the end.



① $f(x) = x^2 \left(\frac{1}{(1-2x)^2} \right)$
 \uparrow $g(x) = \frac{1}{(1-2x)^2}$

③ $g \xrightarrow{\text{mult. by } x^2} f$ $\int g(x) dx = \frac{1/2}{1-2x} + C$ How? See*

$\frac{a=1/2}{r=2x} = \left(\sum_{n=0}^{\infty} \frac{1}{2} (2x)^n \right) + C$
 $= \left(\sum_{n=0}^{\infty} 2^{n-1} x^n \right) + C$

To get $g(x)$,

$g(x) = \frac{d}{dx} \int g(x) dx = \frac{d}{dx} \left[\left(\sum_{n=0}^{\infty} 2^{n-1} x^n \right) + C \right]$
 $= \sum_{n=1}^{\infty} 2^{n-1} n x^{n-1}$ \leftarrow derivative of C is 0

So $g(x) = \sum_{n=1}^{\infty} n 2^{n-1} x^{n-1}$

For f , $f(x) = x^2 g(x) = x^2 \sum_{n=1}^{\infty} n 2^{n-1} x^{n-1}$

$= \sum_{n=1}^{\infty} n 2^{n-1} x^{n+1}$
 for $R = 1/2$

Radius of Convergence?

Use r : $r = 2x$

$|r| = |2x| < 1 \Rightarrow -1 < 2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

Don't check endpoints if only asking for Radius of Convergence.

* $\int \frac{1}{(1-2x)^2} dx$ $u = 1-2x$
 $du = -2dx$
 $= -\frac{1}{2} \int \frac{1}{u^2} du = -\frac{1}{2} \int u^{-2} du = -\frac{1}{2} (-u^{-1}) + C = \frac{1}{2} \cdot \frac{1}{u} + C$
 $= \frac{1}{2} \cdot \frac{1}{(1-2x)} + C = \frac{1/2}{1-2x} + C$

ABG
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